DAY — 08 SEAT NUMBER

2020	II	28	1100	J-360	(E)

MATHEMATICS & STATISTICS (40)
(ARTS & SCIENCE)

Time : 3 Hrs. (8 Pages) Max. Marks : 80

General instructions :

The question paper is divided into ***FOUR*** *sections.*

(1) ***Section A :*** *Q. 1 contains* ***Eight*** *multiple choice type of questions, each carrying* ***Two*** *marks.*
Q. 2 contains ***Four*** *sub-questions, each carrying* ***one*** *mark.*

(2) ***Section B :*** *Q. 3 to Q. 14 each carries* ***Two*** *marks. (Attempt any* ***Eight****)*

(3) ***Section C :*** *Q. 15 to Q. 26 each carries* ***Three*** *marks. (Attempt any* ***Eight****)*

(4) ***Section D :*** *Q. 27 to Q. 34 each carries* ***Four*** *marks. (Attempt any* ***Five****)*

(5) Use of log table is allowed. Use of calculator is not allowed.

(6) Figures to the right indicate full marks.

(7) Use of graph paper is not necessary. Only rough sketch of graph is expected.

(8) For each MCQ, correct answer must be written along with its alphabet:
e.g. (a) .... / (b) .... / (c) .... / (d).........., etc.

(9) Start answers to each section on a new page.

SECTION – A

Q. 1. Select and write the most appropriate answer from the given alternatives for each question : **[16]**

(i) In ΔABC, if $a = 2$, $b = 3$ and $\sin A = \frac{2}{3}$, then $\angle B =$ ______.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$ (2)

(ii) If $\bar{a} = 3\hat{i} - \hat{j} + 4\hat{k}$, $\bar{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\bar{c} = -5\hat{i} + 2\hat{j} + 3\hat{k}$, then $\bar{a} \cdot (\bar{b} \times \bar{c})$ is ______.

(a) 100 (b) 110

(c) 109 (d) 108 (2)

(iii) The cartesian equation of the line passing through the points A (4, 2, 1) and B (2, –1, 3) is ______.

(a) $\frac{x+4}{2} = \frac{y-2}{3} = \frac{z-1}{-2}$

(b) $\frac{x-4}{-2} = \frac{y-2}{-3} = \frac{z-1}{-2}$

(c) $\frac{x-4}{2} = \frac{y-2}{3} = \frac{z-1}{-2}$

(d) $\frac{x-4}{-2} = \frac{y-2}{3} = \frac{z-1}{-2}$ (2)

(iv) If the line $\bar{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ is parallel to the plane $\bar{r} \cdot (3\hat{i} - 2\hat{j} + m\hat{k}) = 10$, then value of m is ______.

(a) – 2 (b) 2

(c) ± 2 (d) 0 (2)

(v) If $f(x) = 1 - x$, for $0 < x \leq 1$

$= k$, for $x = 0$ is continuous at $x = 0$, then $k =$ ______.

(a) 0 (b) -1

(c) 2 (d) 1 (2)

(vi) The function $f(x) = x^x$ is minimum at $x =$ ______.

(a) e (b) $-e$

(c) $\frac{1}{e}$ (d) $-\frac{1}{e}$ (2)

(vii) If $\int_0^k 4x^3 \, dx = 16$, then the value of k is ______.

(a) 1 (b) 2

(c) 3 (d) 4 (2)

(viii) Order and degree of differential equation

$$\frac{d^4y}{dx^4} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3$$ respectively are ______.

(a) Order : 1, Degree : 4

(b) Order : 4, Degree : 1

(c) Order : 6, Degree : 1

(d) Order : 1, Degree : 6 (2)

Q. 2. Answer the following questions : **[4]**

(i) Write the dual of $p \wedge \sim p \equiv F$ (1)

(ii) Find the general solution of $\tan 2x = 0$ (1)

(iii) Differentiate $\sin(x^2 + x)$ w. r. t. x (1)

(iv) If $X \sim B(n, p)$ and $n = 10$, $E(X) = 5$, then find the value of p. (1)

SECTION – B

Attempt any EIGHT of the following questions : [16]

Q. 3. Using truth table verify that $\sim(p \vee q) \equiv \sim p \wedge \sim q$ (2)

Q. 4. Find the matrix of co-factors for the matrix $\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$ (2)

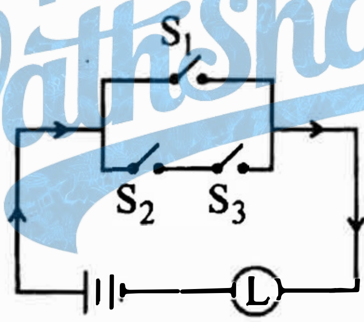
Q. 5. Find the angle between the lines represented by

$3x^2 + 4xy - 3y^2 = 0$ (2)

Q. 6. $\bar{a}$ and $\bar{b}$ are non-collinear vectors. If $\bar{c} = (x-2)\bar{a} + \bar{b}$ and $\bar{d} = (2x+1)\bar{a} - \bar{b}$ are collinear, then find the value of x. (2)

Q. 7. If a line makes angles 90°, 135°, 45° with X, Y and Z axes respectively, then find its direction cosines. (2)

Q. 8. Express the following circuit in symbolic form : (2)



Q. 9. Differentiate $\log(\sec x + \tan x)$ w. r. t. x. (2)

Q. 10. Evaluate : $\int \frac{dx}{x^2 + 4x + 8}$ (2)

Q. 11. Evaluate : $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$ (2)

Q. 12. Solve the differential equation $\frac{dy}{dx} = x^2y + y$ (2)

Q. 13. Find expected value of the random variable X whose probability mass function is :

$X = x$	1	2	3
$P(X = x)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

(2)

Q. 14. If $y = x \log x$, then find $\frac{d^2y}{dx^2}$. (2)

SECTION – C

Attempt any EIGHT of the following questions : **[24]**

Q. 15. State the converse, inverse and contrapositive of the conditional statement : 'If a sequence is bounded, then it is convergent'. (3)

Q. 16. Show that : $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{77}{85}\right)$. (3)

Q. 17. Show that the points A(2, 1, –1), B(0, –1, 0), C(4, 0, 4) and D(2, 0, 1) are coplanar. (3)

Q. 18. If ΔABC is right angled at B, where A(5, 6, 4), B(4, 4, 1) and C(8, 2, x), then find the value of x. (3)

Q. 19. Find the equation of the line passing through the point (3, 1, 2) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$. (3)

Q. 20. Find the distance of the point $\hat{i}+2\hat{j}-\hat{k}$ from the plane $\bar{r}\cdot(\hat{i}-2\hat{j}+4\hat{k})=10$ (3)

Q. 21. If $e^x+e^y=e^{x+y}$, show that $\frac{dy}{dx}=-e^{y-x}$ (3)

Q. 22. The surface area of a spherical balloon is increasing at the rate of 2 cm^2/sec. At what rate the volume of the balloon is increasing when the radius of the balloon is 6 cm? (3)

Q. 23. Find the approximate value of $e^{1\cdot005}$; given $e=2\cdot7183$. (3)

Q. 24. Evaluate :

$$\int\frac{x^2\cdot\tan^{-1}(x^3)}{1+x^6}dx$$ (3)

Q. 25. Solve the differential equation $\frac{dy}{dx}+y=e^{-x}$ (3)

Q. 26. If $f(x)=kx,\ 0<x<2$

$=0$, otherwise,

is a probability density function of a random variable X, then find :

(i) Value of k,

(ii) $P(1<X<2)$. (3)

SECTION – D

Attempt any FIVE of the following questions : **[20]**

Q. 27. Prove that a homogeneous equation of degree two in x and y i.e. $ax^2+2hxy+by^2=0$ represents a pair of lines passing through the origin, if $h^2-ab\geq0$. (4)

Q. 28. Solve the following linear programming problem :

Maximise : $z = 150x + 250y$

Subject to ; $4x + y \leq 40$

$3x + 2y \leq 60$

$x \geq 0$

$y \geq 0$ (4)

Q. 29. Solve the following equations by the method of reduction :

$x + 3y + 3z = 12$

$x + 4y + 4z = 15$

$x + 3y + 4z = 13$ (4)

Q. 30. In Δ ABC, if $a + b + c = 2s$, then prove that

$\sin\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{bc}}$, with usual notations. (4)

Q. 31. Function $f(x)$ is continuous on its domain [–2, 2], where

$f(x) = \frac{\sin ax}{x} + 2$, for $-2 \leq x < 0$

$= 3x + 5$, for $0 \leq x \leq 1$

$= \sqrt{x^2 + 8} - b$, for $1 < x \leq 2$

Find the value of $a + b + 2$. (4)

Q. 32. Prove that :

$$\int \sqrt{x^2 + a^2} \cdot dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$$ (4)

Q. 33. A fair coin is tossed 8 times. Find the probability that :

(i) it shows no head

(ii) it shows head at least once. (4)

Q. 34. Prove that :

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx \qquad (4)$$

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