

--	--	--	--	--	--

2019 III 02

1100

J - 60

(E)

MATHEMATICS & STATISTICS (40)

(ARTS & SCIENCE)

Time : 3 Hrs.

(7 Pages)

Max. Marks : 80

- Note :**
- (1) All questions are compulsory.
 - (2) Figures to the right indicate full marks.
 - (3) The question paper consists of 30 questions divided into **FOUR** sections A, B, C, D.
 - Section A contains 6 questions of 1 mark each.
 - Section B contains 8 questions of 2 marks each. (One of them has internal option)
 - Section C contains 6 questions of 3 marks each. (Two of them have internal options)
 - Section D contains 10 questions of 4 marks each. (Three of them have internal options)
 - (4) For each MCQ, correct answer must be written along with its **alphabet**,
 e.g. (a)..... / (b)..... / (c)..... / (d).....etc.
 In case of MCQ (Q. No. 1. to 6.) evaluation would be done for the first attempt only.
 - (5) Use of logarithmic table is allowed. Use of calculator is **not** allowed.
 - (6) In L. P. P. only rough sketch of graph is expected. Graph paper is **not** necessary.
 - (7) Start each section on new page only.

SECTION – A

Select and write the most appropriate answer from the given alternatives for each question : [6]

Q. 1. The principal solutions of $\cot x = -\sqrt{3}$ are _____. (1)

- (a) $\frac{\pi}{6}, \frac{5\pi}{6}$ (b) $\frac{5\pi}{6}, \frac{7\pi}{6}$
(c) $\frac{5\pi}{6}, \frac{11\pi}{6}$ (d) $\frac{\pi}{6}, \frac{11\pi}{6}$

Q. 2. The acute angle between the two planes $x + y + 2z = 3$ and $3x - 2y + 2z = 7$ is _____. (1)

- (a) $\sin^{-1}\left(\frac{5}{\sqrt{102}}\right)$ (b) $\cos^{-1}\left(\frac{5}{\sqrt{102}}\right)$
(c) $\sin^{-1}\left(\frac{15}{\sqrt{102}}\right)$ (d) $\cos^{-1}\left(\frac{15}{\sqrt{102}}\right)$

Q. 3. The direction ratios of the line which is perpendicular to the lines with direction ratios $-1, 2, 2$ and $0, 2, 1$ are _____. (1)

- (a) $-2, -1, -2$ (b) $2, 1, 2$
(c) $2, -1, -2$ (d) $-2, 1, -2$

Q. 4. If $f(x) = (1 + 2x)^{\frac{1}{x}}$, for $x \neq 0$ is continuous at $x = 0$, then $f(0) =$ _____. (1)

- (a) e (b) e^2
(c) 0 (d) 2

Q. 5. $\int \frac{dx}{9x^2 + 1} = \underline{\hspace{2cm}}$. (1)

(a) $\frac{1}{3} \tan^{-1}(2x) + c$

(b) $\frac{1}{3} \tan^{-1} x + c$

(c) $\frac{1}{3} \tan^{-1}(3x) + c$

(d) $\frac{1}{3} \tan^{-1}(6x) + c$

Q. 6. If $y = ae^{5x} + be^{-5x}$, then the differential equation is _____. (1)

(a) $\frac{d^2y}{dx^2} = 25y$

(b) $\frac{d^2y}{dx^2} = -25y$

(c) $\frac{d^2y}{dx^2} = -5y$

(d) $\frac{d^2y}{dx^2} = 5y$

SECTION – B

Q. 7. Write the truth values of the following statements : [16]

(i) 2 is a rational number and $\sqrt{2}$ is an irrational number.

(ii) $2 + 3 = 5$ or $\sqrt{2} + \sqrt{3} = \sqrt{5}$ (2)

Q. 8. Find the volume of the parallelopiped, if the coterminal edges are given by the vectors $2\hat{i} + 5\hat{j} - 4\hat{k}$, $5\hat{i} + 7\hat{j} + 5\hat{k}$, $4\hat{i} + 5\hat{j} - 2\hat{k}$. (2)

OR

Find the value of p , if the vectors $\hat{i} - 2\hat{j} + \hat{k}$, $2\hat{i} - 5\hat{j} + p\hat{k}$ and $5\hat{i} - 9\hat{j} + 4\hat{k}$ are coplanar.

Q. 9. Show that the points A(-7, 4, -2), B(-2, 1, 0) and C(3, -2, 2) are collinear. (2)

Q. 10. Write the equation of the plane $3x + 4y - 2z = 5$ in the vector form. (2)

Q. 11. If $y = x^x$, find $\frac{dy}{dx}$. (2)

Q. 12. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at $(-1, -2)$. (2)

Q. 13. Evaluate: $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ (2)

Q. 14. Evaluate: $\int_0^{\frac{\pi}{2}} \sin^2 x dx$ (2)

SECTION - C

Q. 15. In $\triangle ABC$, prove that [18]

$$\sin\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{a}\right) \cos\left(\frac{A}{2}\right) \quad (3)$$

OR

Show that $\sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{63}{16}\right)$

Q. 16. If $A(\vec{a})$ and $B(\vec{b})$ are any two points in the space and $R(\vec{r})$ be a point on the line segment AB dividing it internally in the ratio $m : n$, then prove that

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n} \quad (3)$$

Q. 17. The equation of a line is $2x - 2 = 3y + 1 = 6z - 2$, find its direction ratios and also find the vector equation of the line. (3)

Q. 18. Discuss the continuity of the function

$$f(x) = \frac{\log(2+x) - \log(2-x)}{\tan x}, \text{ for } x \neq 0$$

$$= 1 \quad \text{for } x = 0$$

at the point $x = 0$ (3)

Q. 19. The probability distribution of a random variable X , the number of defects per 10 meters of a fabric is given by

x	0	1	2	3	4
$P(X = x)$	0.45	0.35	0.15	0.03	0.02

Find the variance of X . (3)

OR

For the following probability density function (p. d. f.) of X , find : (i) $P(X < 1)$, (ii) $P(|X| < 1)$

$$\text{if } f(x) = \frac{x^2}{18}, \quad -3 < x < 3$$

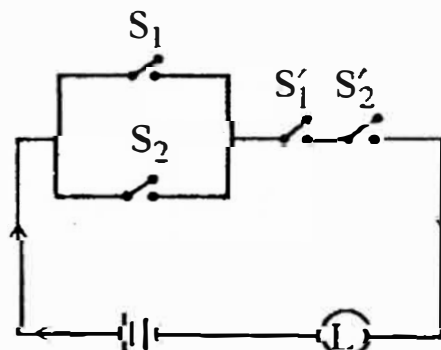
$$= 0, \quad \text{otherwise}$$

Q. 20. Given is $X \sim B(n, p)$.

If $E(X) = 6$, $\text{Var.}(X) = 4.2$, find n and p . (3)

SECTION – D

Q. 21. Find the symbolic form of the given switching circuit. Construct its switching table and interpret your result. [40]



(4)

0 0 6 0

Q. 22. If three numbers are added, their sum is 2. If two times the second number is subtracted from the sum of first and third numbers we get 8 and if three times the first number is added to the sum of second and third numbers we get 4. Find the numbers using matrices. (4)

Q. 23. In ΔABC , with usual notations prove that

$$b^2 = c^2 + a^2 - 2ca \cos B \quad (4)$$

OR

In ΔABC , with usual notations prove that

$$(a-b)^2 \cos^2\left(\frac{C}{2}\right) + (a+b)^2 \sin^2\left(\frac{C}{2}\right) = c^2.$$

Q. 24. Find 'p' and 'q' if the equation

$$px^2 - 8xy + 3y^2 + 14x + 2y + q = 0 \text{ represents a pair of perpendicular lines.} \quad (4)$$

Q. 25. Maximize :

$$\begin{aligned} z &= 3x + 5y && \text{Subject to} \\ x + 4y &\leq 24, && 3x + y \leq 21, \\ x + y &\leq 9, && x \geq 0, y \geq 0 \end{aligned} \quad (4)$$

Q. 26. If $x = f(t)$ and $y = g(t)$ are differentiable functions of t , then prove that y is a differentiable function of x and

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \text{ where } \frac{dx}{dt} \neq 0$$

$$\text{Hence find } \frac{dy}{dx} \text{ if } x = a \cos^2 t \text{ and } y = a \sin^2 t. \quad (4)$$

Q. 27. $f(x) = (x-1)(x-2)(x-3)$, $x \in [0,4]$, find 'c' if LMVT can be applied. (4)

OR

A rod of 108 meters long is bent to form a rectangle. Find its dimensions if the area is maximum.

Q. 28. Prove that : $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right| + c$ (4)

Q. 29. Show that : $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx = \frac{\pi}{8} \log 2$ (4)

Q. 30. Solve the differential equation :

$$\frac{dy}{dx} + y \sec x = \tan x$$
 (4)

OR

Solve the differential equation :

$$(x+y) \frac{dy}{dx} = 1$$

