DAY —— 08 SEAT NUMBER

2018 | III | 03 | 1100 | **J - 265** | (E)

MATHEMATICS & STATISTICS (40)
(ARTS & SCIENCE)

Time : 3 Hrs. (7 Pages) **Max. Marks : 80**

Note : *(i)* *All questions are compulsory.*

(ii) *Figures to the right indicate full marks.*

(iii) *Graph of L.P.P. should be drawn on graph paper only.*

(iv) *Use of logarithmic table is allowed.*

(v) *Answers to the questions of Section - I and Section - II should be written in only one answer book.*

(vi) *Answer to every new question must be written on a new page.*

SECTION – I

Q. 1. **(A)** Select and write the appropriate answer from the given alternatives in each of the following sub-questions : **[12]** **(6)**

(i) If $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$, then adjoint of matrix A is _____.

(a) $\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$ (2)

(ii) The principal solutions of $\sec x = \frac{2}{\sqrt{3}}$ are ______.

(a) $\frac{\pi}{3}, \frac{11\pi}{6}$ (b) $\frac{\pi}{6}, \frac{11\pi}{6}$

(c) $\frac{\pi}{4}, \frac{11\pi}{4}$ (d) $\frac{\pi}{6}, \frac{11\pi}{4}$ (2)

(iii) The measure of acute angle between the lines whose direction ratios are 3, 2, 6 and −2, 1, 2 is ______.

(a) $\cos^{-1}\left(\frac{1}{7}\right)$ (b) $\cos^{-1}\left(\frac{8}{15}\right)$

(c) $\cos^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{8}{21}\right)$ (2)

(B) Attempt any THREE of the following : **(6)**

(i) Write the negations of the following statements :

(a) All students of this college live in the hostel.

(b) 6 is an even number or 36 is a perfect square. (2)

(ii) If a line makes angles α, β, γ with the co-ordinate axes, prove that $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + 1 = 0$. (2)

(iii) Find the distance of the point (1, 2, −1) from the plane $x - 2y + 4z - 10 = 0$. (2)

(iv) Find the vector equation of the line which passes through the point with position vector $4\hat{i} - \hat{j} + 2\hat{k}$ and is in the direction of $-2\hat{i} + \hat{j} + \hat{k}$. (2)

(v) If $\bar{a} = 3\hat{i} - 2\hat{j} + 7\hat{k}$, $\bar{b} = 5\hat{i} + \hat{j} - 2\hat{k}$ and $\bar{c} = \hat{i} + \hat{j} - \hat{k}$, then find $\bar{a} \cdot (\bar{b} \times \bar{c})$. (2)

Q. 2. (A) Attempt any TWO of the following : (6) [14]

(i) Using vector method prove that the medians of a triangle are concurrent. (3)

(ii) Using the truth table, prove the following logical equivalence :

$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$. (3)

(iii) If the origin is the centroid of the triangle whose vertices are $A(2, p, -3)$, $B(q, -2, 5)$ and $R(-5, 1, r)$, then find the values of p, q, r. (3)

(B) Attempt any TWO of the following : (8)

(i) Show that a homogeneous equation of degree two in x and y, i.e. $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin if $h^2 - ab \geq 0$. (4)

(ii) In ΔABC, prove that $\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right)\cot\frac{B}{2}$ (4)

(iii) Find the inverse of the matrix, $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ using elementary row transformations. (4)

Q. 3. (A) Attempt any TWO of the following : (6) [14]

(i) Find the joint equation of the pair of lines passing through the origin, which are perpendicular to the lines represented by $5x^2 + 2xy - 3y^2 = 0$. (3)

(ii) Find the angle between the lines $\frac{x-1}{4}=\frac{y-3}{1}=\frac{z}{8}$ and

$\frac{x-2}{2}=\frac{y+1}{2}=\frac{z-4}{1}$ (3)

(iii) Write converse, inverse and contrapositive of the following conditional statement :

If an angle is a right angle then its measure is 90°. (3)

(B) Attempt any TWO of the following : **(8)**

(i) Prove that : $\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\sin^{-1}\left(\frac{56}{65}\right)$ (4)

(ii) Find the vector equation of the plane passing through the points A(1, 0, 1), B(1, –1, 1) and C(4, –3, 2). (4)

(iii) Minimize $Z = 7x + y$ subject to

$5x + y \geq 5,\ x + y \geq 3,\ x \geq 0,\ y \geq 0$ (4)

SECTION - II

Q. 4. **(A)** Select and write the appropriate answer from the given alternatives in each of the following sub-questions : **[12]** **(6)**

(i) Let the p. m. f. of a random variable X be –

$P(x)=\frac{3-x}{10}$ for $x = -1, 0, 1, 2$

$= 0$ otherwise

Then E(X) is ______.

(a) 1 (b) 2

(c) 0 (d) –1 (2)

(ii) If $\int_0^k \frac{1}{2+8x^2}\,dx = \frac{\pi}{16}$, then the value of k is ______.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$ (2)

(iii) Integrating factor of linear differential equation

$x\frac{dy}{dx} + 2y = x^2 \log x$ is ______.

(a) $\frac{1}{x^2}$ (b) $\frac{1}{x}$

(c) x (d) x^2 (2)

(B) Attempt any THREE of the following : **(6)**

(i) Evaluate : $\int e^x\left[\frac{\cos x - \sin x}{\sin^2 x}\right]dx$ (2)

(ii) If $y = \tan^2(\log x^3)$, find $\frac{dy}{dx}$. (2)

(iii) Find the area of ellipse $\frac{x^2}{1} + \frac{y^2}{4} = 1$. (2)

(iv) Obtain the differential equation by eliminating the arbitrary constants from the following equation :

$y = c_1 e^{2x} + c_2 e^{-2x}$ (2)

(v) Given $X \sim B(n, p)$

If $n = 10$ and $p = 0{\cdot}4$, find $E(X)$ and $Var.(X)$. (2)

Q. 5. (A) Attempt any TWO of the following : **(6) [14]**

(i) Evaluate : $\int \frac{1}{3+2\sin x+\cos x}\,dx$ (3)

(ii) If $x = a\cos^3 t$, $y = a\sin^3 t$,

show that $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$ (3)

(iii) Examine the continuity of the function :

$$f(x) = \frac{\log 100 + \log(0\cdot 01 + x)}{3x}, \text{ for } x \neq 0$$

$$= \frac{100}{3} \qquad \text{for } x = 0; \text{ at } x = 0$$ (3)

(B) Attempt any TWO of the following : **(8)**

(i) Find the maximum and minimum value of the function :

$f(x) = 2x^3 - 21x^2 + 36x - 20.$ (4)

(ii) Prove that : $\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$ (4)

(iii) Show that :

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx, \text{ if } f(x) \text{ is an even function.}$$

$$= 0, \qquad \text{if } f(x) \text{ is an odd function.}$$ (4)

Q. 6. (A) Attempt any TWO of the following : **(6) [14]**

(i) If $f(x) = \dfrac{x^2 - 9}{x - 3} + \alpha, \quad$ for $x > 3$

$= 5 \quad$, for $x = 3$

$= 2x^2 + 3x + \beta, \quad$ for $x < 3$

is continuous at $x = 3$, find α and β. (3)

(ii) Find $\dfrac{dy}{dx}$ if $y = \tan^{-1}\left(\dfrac{5x + 1}{3 - x - 6x^2}\right)$ (3)

(iii) A fair coin is tossed 9 times. Find the probability that it shows head exactly 5 times. (3)

(B) Attempt any TWO of the following : **(8)**

(i) Verify Rolle's theorem for the following function :

$f(x) = x^2 - 4x + 10$ on $[0, 4]$ (4)

(ii) Find the particular solution of the differential equation :

$y(1 + \log x)\dfrac{dx}{dy} - x \log x = 0$

when $y = e^2$ and $x = e$. (4)

(iii) Find the variance and standard deviation of the random variable X whose probability distribution is given below :

x	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(4)

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