

SECTION-A

Q1.

$$x^2 \frac{d^2 y}{dx^2} = \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^4$$

ORDER = 2

DEGREE = 1

Q2.

$$f(x) = x+7 ; g(x) = x-7$$

$$f \circ g(x) = f(g(x))$$

$$= f(x-7)$$

$$= (x-7)+7$$

$$= x$$

$$\forall x \in \mathbb{R}$$

$$\frac{d}{dx} f \circ g(x) = \frac{d}{dx} (x) = 1$$

$$\boxed{\frac{d}{dx} f \circ g(x) = 1}$$

Q3

$$2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Comparing corresponding elements of each matrix,

$$2+y=5$$

$$\boxed{y=3}$$

$$2x+2=8$$

$$\boxed{x=3}$$

$$x-y=3-3$$

$$\boxed{x-y=0}$$

Q4

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$$

where $\vec{a}$ = position vector of pt
on line
 $\vec{b}$ = parallel vector on line

Also, $\vec{r} = (3+2\lambda)\hat{i} + (4+2\lambda)\hat{j} + (5-3\lambda)\hat{k}$

Q5.

$$a * b = ab + 1$$

i) For all $(a, b) \in (R \times R)$

$$\Rightarrow ab + 1 \in R \quad [\because \text{If } a, b \in R \Rightarrow ab \in R \Rightarrow (ab + 1) \in R] \quad [\because \text{Multiplication is binary operation}]$$

$\therefore a R b$ relates to a unique element in R

and hence is a binary operation from $R \times R$ to R

ii) For binary operation to be associative $(a * b) * c = a * (b * c)$

$$\text{LHS} = (a * b) * c$$

$$= (ab + 1) * c$$

$$= (ab + 1)c + 1$$

$$= abc + c + 1$$

$$\text{RHS} = a * (b * c)$$

$$= a * (bc + 1)$$

$$= a(bc + 1) + 1$$

$$= abc + a + 1$$

$$\text{LHS} \neq \text{RHS}$$

$\therefore$ It is NOT ASSOCIATIVE

Q6

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$A^2 = AA$$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$A^2 - 5A = A^2 + (-5)A$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + (-5) \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}$$

$$A^2 - 5A = A^2 + (-5A)$$

$$= \begin{bmatrix} +5 & -7 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix}$$

Q7. $I = \int \frac{1. \sin^{-1}(2x)}{x} dx$

$I = [\sin^{-1}(2x)](x) - \int \frac{(x)(2)}{\sqrt{1-4x^2}} dx$ [INTEGRATION by parts]

$$I = x \sin^{-1}(2x) - \frac{1}{2} \int \frac{2x}{\sqrt{(\frac{1}{2})^2 - x^2}} dx$$

$$I = x \sin^{-1}(2x) + \frac{1}{4} \int \frac{(-8x)}{\sqrt{1-4x^2}} dx = x \sin^{-1}(2x) + \frac{1}{4} I_1 \quad \text{where } I_1 = \int \frac{-8x}{\sqrt{1-4x^2}} dx$$

Now, $I_1 = \int \frac{dt}{\sqrt{t}}$

let $1-4x^2 = t$

$-8x dx = dt$

$$= 2\sqrt{t} + C$$

$$= 2\sqrt{1-4x^2} + C$$

$$I = x \sin^{-1}(2x) + \frac{1}{4} (2\sqrt{1-4x^2}) + C$$

$$I = x \sin^{-1}(2x) + \frac{\sqrt{1-4x^2}}{2} + C$$

Q6

$$y = e^{2x} (a+bx) \quad \text{--- (1)}$$

Diff. both sides w.r.t x .

$$\frac{dy}{dx} = e^{2x} (b) + (a+bx)(e^{2x})(2)$$

$$\Rightarrow \frac{dy}{dx} = e^{2x} (b+2a+2bx) \quad \text{--- (2)}$$

Diff. both sides w.r.t x .

$$\frac{d^2y}{dx^2} = e^{2x} (2b) + (b+2a+2bx)(e^{2x})(2)$$

$$= e^{2x} (4b+4a+4bx)$$

$$\frac{d^2y}{dx^2} = e^{2x} (2b+2a+2bx) \quad \text{--- (3)}$$

Subtract ^{eqn} $2x(2)$ from eqn(3)

$$\frac{d^2y}{dx^2} - \frac{2dy}{dx} = e^{2x} (4b+4a+4bx) - e^{2x} (2bx+4a+4bx)$$

$$= e^{2x} (2b)$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 2(e^{2x})(b) \quad \text{--- (4)}$$

In eqⁿ (3)

$$\frac{d^2y}{dx^2} = e^{2x}(4b + 4a + 4bx)$$

$$= 2\left(\frac{d^2y}{dx^2} - 2\frac{dy}{dx}\right) + 4e^{2x}(a + bx) \quad [\text{From (4)}]$$

$$\frac{d^2y}{dx^2} = 2\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y \quad [\text{From (1)}]$$

∴ The required differential eqⁿ is

$$\boxed{\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0} \Rightarrow y'' - 4y' + 4y = 0.$$

where $y'' = \frac{d^2y}{dx^2}$, $y' = \frac{dy}{dx}$, $y = e^2$

R.T.O

Q9

$$\sum_{i=0}^2 P(X_i) = 1$$

$$\Rightarrow k + 2k + 3k = 1$$

$$\Rightarrow 6k = 1$$

$$\boxed{k = \frac{1}{6}}$$

$$\left[\because \sum_{i=0}^2 P(X_i) = 1 \right]$$

Sum of all probabilities = 1
disjoint exhaustive
exclusive

Q11

Q10

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

$$P(B) = \frac{3}{6} = \frac{1}{2}$$

$$P(A \cap B) = P(\text{no. which is even and red}) = \frac{1}{6}$$

$$P(E) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of possible outcomes}}$$

$$\left[\begin{aligned} \Rightarrow \text{Sample space } S &= \{1r, 2r, 3r, 4g, 5g, 6g\} \\ A &= \{2r, 4g, 6g\} & A \cap B &= \{2r\} \\ B &= \{1r, 2r, 3r\} \end{aligned} \right]$$

$$\left[\because \text{No. which is even and red} = \{2r\} \right]$$

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(A \cap B) \neq P(A) \cdot P(B)$$

$\therefore$ Events are ~~not~~ NOT INDEPENDENT.

Q12

Q11 $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$

$[\vec{a} \vec{b} \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$

$= \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k} = -3\hat{i} + \hat{j} + 2\hat{k}$

$= \begin{bmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{bmatrix}$

$= 2(-4-1) - 3(2+3) + 1(1-6)$

$= -10 - 15 - 5$

$\therefore [\vec{a} \vec{b} \vec{c}] = -30$

Q12 $I = \int \frac{\tan^2 x \cdot \sec^2 x}{1 - \tan^6 x} dx$

let $\tan^3 x = t$

$3\tan^2 x \sec^2 x dx = dt$

$$I = \int \frac{t^2 dt}{1+t^6} = \int \frac{t^2 dt}{1+(t^3)^2}$$

$$\text{let } t^3 = u \\ 3t^2 dt$$

$$I = \frac{1}{3} \int \frac{du}{u^2 + 1}$$

$$= \frac{1}{3} \cdot \frac{1}{2 \cdot (1)} \log \left| \frac{1+u}{1-u} \right| + C \quad \because \int \frac{du}{a^2 - u^2} = \log \left| \frac{a+u}{a-u} \right| + C$$

$$I = \frac{1}{6} \log \left| \frac{1+\tan^3 x}{1-\tan^3 x} \right| + C$$

SECTION-C

P.T.O

Q13. $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

$$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right), AB < 1$$

$$\Rightarrow \tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}$$

taking tan on both side
 $\Rightarrow \frac{5x}{1-6x^2} = 1$

$$\Rightarrow 6x^2 + 5x - 1 = 0$$

$$\Rightarrow 6x^2 + 6x - x - 1 = 0$$

$$\Rightarrow (6x-1)(x+1) = 0$$

$$\Rightarrow x = \frac{1}{6} \text{ or } x = -1$$

and $(2x)(3x) < 1$

$$6x^2 < 1$$

$$-\frac{1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$$

$\therefore x = -1$ is not a solution $[\because -1 \notin (-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}})]$
 $\Rightarrow \boxed{x = \frac{1}{6}}$ is the solⁿ.

VERIFICATION

For $x = \frac{1}{6}$

$$\text{LHS: } \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}\left(\frac{5}{1}\right) = \frac{\pi}{4} = \text{RHS.}$$

$$\text{LHS} = \text{RHS}$$

$$\Rightarrow x = \frac{1}{6} \text{ is a sol}^n$$

$$\text{For } x = -1$$

$$\tan^{-1}(-2) - \tan^{-1}(-3) = \frac{\pi - \cot^{-1}(-2) - \pi + \cot^{-1}(-3)}{2}$$

$$= \pi - \cot^{-1}(3) - \pi + \cot^{-1}(2)$$

$$= \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2} - \frac{1}{3}}{1 + \frac{1}{2} \cdot \frac{1}{3}}\right) = \tan^{-1}\left(\frac{1}{7}\right) \neq \text{RHS}$$

LHS $\neq$ RHS

$\therefore x = -1$ is NOT solⁿ

Q14

$$\log(x^2 + y^2) = 2 \tan^{-1} \frac{y}{x}$$

Diff. both sides w.r.t x

$$\frac{1}{x^2 + y^2} (2x + 2y \frac{dy}{dx}) = \frac{2}{1 + \frac{y^2}{x^2}} \cdot \frac{d}{dx} \left(\frac{y}{x} \right)$$

$$\left[\because \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2} ; \frac{d}{dx} (\log x) = \frac{1}{x} \right]$$

$$\Rightarrow \frac{x + y y'}{x^2 + y^2} = \frac{x^2}{x^2 + y^2} \cdot \frac{(x y' - y)}{x^2}$$

$$[\text{where } y' = \frac{dy}{dx}]$$

$$\Rightarrow x + y y' = x y' - y$$

$$\Rightarrow x + y = y'(y - x)$$

$$\Rightarrow y' = \frac{x+y}{x-y}$$

$$\boxed{\frac{dy}{dx} = \frac{x+y}{x-y}}$$

Hence proved.

Q15 $I = \int \frac{3x+5}{x^2+3x-18} dx$

$$3x+5 = a \left[\frac{d}{dx} (x^2+3x-18) \right] + b$$

$$= a(2x+3) + b$$

comparing coefficients on both sides,

$$3 = 2a$$

$$\boxed{a = \frac{3}{2}}$$

$$3a+b=5$$

$$b = 5 - 3\left(\frac{3}{2}\right) = \frac{1}{2}$$

$$\Rightarrow \boxed{b = \frac{1}{2}}$$

P.T.O

$$I = \int \frac{\frac{3}{2}(2x+3) + \frac{1}{2}}{x^2+3x-18} dx$$

$$= \frac{3}{2} \int \frac{(2x+3)}{x^2+3x-18} dx + \frac{1}{2} \int \frac{dx}{x^2+3x-18}$$

$$= \frac{3}{2} I_1 + \frac{1}{2} I_2$$

$$I_1 = \int \frac{2x+3}{x^2+3x-18} dx$$

$$\text{Let } x^2+3x-18 = t$$

$$(2x+3)dx = dt$$

$$I_1 = \int \frac{dt}{t} = \log|t| + c_1$$

$$I_1 = \log|x^2+3x-18| + c_1$$

$$\begin{aligned} \text{Ans, } I_2 &= \int \frac{dx}{x^2+3x-18} = \int \frac{dx}{x^2+3x+\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 18 - \left(\frac{3}{2}\right)^2} \\ &= \int \frac{dx}{\left(x+\frac{3}{2}\right)^2 - 18 - \frac{9}{4}} = \int \frac{dx}{\left(x+\frac{3}{2}\right)^2 - \frac{81}{4}} \end{aligned}$$

$$I_1 = - \int_a^0 f(a-t) dt$$

 $\Rightarrow$

$$= \int_0^a f(a-t) dt \quad \left[\because \int_a^b f(x) dx = - \int_b^a f(x) dx \right]$$

$$I_1 = \int_0^a f(a-x) dx \quad \left[\because \int_a^b f(t) dt = \int_a^b f(x) dx \right]$$

 $\Rightarrow$

LHS = RHS

Hence proved

$$I = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx \quad \text{--- (1)}$$

$$I = \int_0^{\pi} \frac{(\pi-x) \sin(\pi-x)}{1 + \cos^2(\pi-x)} dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$I = \int_0^{\pi} \frac{(\pi-x) \sin x}{1 + \cos^2 x} dx \quad \text{--- (2)}$$

Adding eqn (1), (2)

$$2I = \int_0^{\pi} \frac{\pi \sin x}{1 + \cos^2 x} dx$$

$$\begin{aligned}
 I_2 &= \int \frac{dx}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} \\
 &= \frac{1}{2 \cdot \frac{9}{2}} \log \left| \frac{\left(x+\frac{3}{2}\right) - \frac{9}{2}}{\left(x+\frac{3}{2}\right) + \frac{9}{2}} \right| + C_2 \\
 &= \frac{1}{9} \log \left| \frac{x-3}{x+6} \right| + C_2
 \end{aligned}$$

$$I = \frac{3}{2} \log |x^2 + 3x - 18| + C_1 + \frac{1}{18} \log \left| \frac{x-3}{x+6} \right| + C_2$$

$$I = \frac{3}{2} \log |x^2 + 3x - 18| + \frac{1}{18} \log \left| \frac{x-3}{x+6} \right| + C \quad \text{where } C = C_1 + C_2 = \text{const.}$$

Q16

$$\text{T.P: } \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$\text{LHS: } I_1 = \int_0^a f(x) dx$$

$$\begin{aligned} \text{let } x &= a-t \\ dx &= -dt \end{aligned}$$

$$\begin{aligned} \text{When } x=0 & \quad t=a \\ x=a & \quad t=0 \end{aligned}$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{\sin x \, dx}{1 + \cos^2 x}$$

$$\text{let } \cos x = t$$

$$-\sin x \, dx = dt$$

$$\text{When } x=0 \quad t=1$$

$$x=\pi \quad t=-1$$

$$\Rightarrow I = \frac{\pi}{2} \int_1^{-1} \frac{-dt}{1+t^2}$$

$$= -\frac{\pi}{2} [\tan^{-1} t]_1^{-1} = -\frac{\pi}{2} \left[-\frac{\pi}{4} - \frac{\pi}{4} \right] = \frac{\pi}{2} \cdot \frac{\pi}{2}$$

$$\boxed{I = \frac{\pi^2}{4}}$$

$$\therefore \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} = \frac{\pi^2}{4}$$

Q17

$$\vec{A} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{B} = 2\hat{i} + 5\hat{j} + 0\hat{k}$$

$$\vec{C} = 3\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{D} = \hat{i} - 6\hat{j} - \hat{k}$$

$\vec{AB}$ = Position vector of B - Position vector of A

$$\vec{m} = \hat{i} + 4\hat{j} - \hat{k}$$

$\vec{CD}$ = Position vector of D - Position vector of C

$$\vec{n} = -2\hat{i} - 8\hat{j} + 2\hat{k}$$

Angle b/w $\vec{AB}$ and $\vec{CD} = \theta$

$$\cos \theta = \frac{\vec{m} \cdot \vec{n}}{|\vec{m}| |\vec{n}|}$$

$$= \frac{m_1 n_1 + m_2 n_2 + m_3 n_3}{\sqrt{m_1^2 + m_2^2 + m_3^2} \sqrt{n_1^2 + n_2^2 + n_3^2}}$$

$$= \frac{-2 - 32 - 2}{\sqrt{1+16+1} \sqrt{4+64+4}}$$

$$= \frac{-36}{\sqrt{18} \sqrt{72}}$$

$$= \frac{-36}{(3\sqrt{2})(6\sqrt{2})} = \frac{-36}{18 \times 2}$$

$$= \frac{-36}{36} = -1$$

$$\theta = \cos^{-1}(-1) \Rightarrow \theta = \pi$$

$\therefore$ Since, $\vec{AB}$ and $\vec{CD}$ are antiparallel,
they ARE COLLINEAR

Q18.

$$\Delta = \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

$$\Delta = \begin{vmatrix} a+b & a+b & -(a+b) \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

$$R_1 \rightarrow R_1 + R_2$$

$$= (a+b) \begin{vmatrix} 1 & 1 & -1 \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

Taking $(a+b)$ common from R_1

$$= (a+b) \begin{vmatrix} 1 & 1 & -1 \\ -(b+c) & b+c & b+c \\ -b & -a & a+b+c \end{vmatrix}$$

$$R_2 \rightarrow R_2 + R_3$$

$$= (a+b)(b+c) \begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -b & -a & a+b+c \end{vmatrix}$$

Taking $(b+c)$ common from R_2

$$= (a+b)(b+c) \begin{vmatrix} 0 & 0 & -1 \\ 0 & 2 & 1 \\ a+c & b+c & a+b+c \end{vmatrix}$$

$$C_1 \rightarrow C_1 + C_3$$

$$C_2 \rightarrow C_2 + C_3$$

$$\Delta = (a+b)(b+c)(c+a)$$

0	0	-1
0	2	1
1	b+c	a+b+c

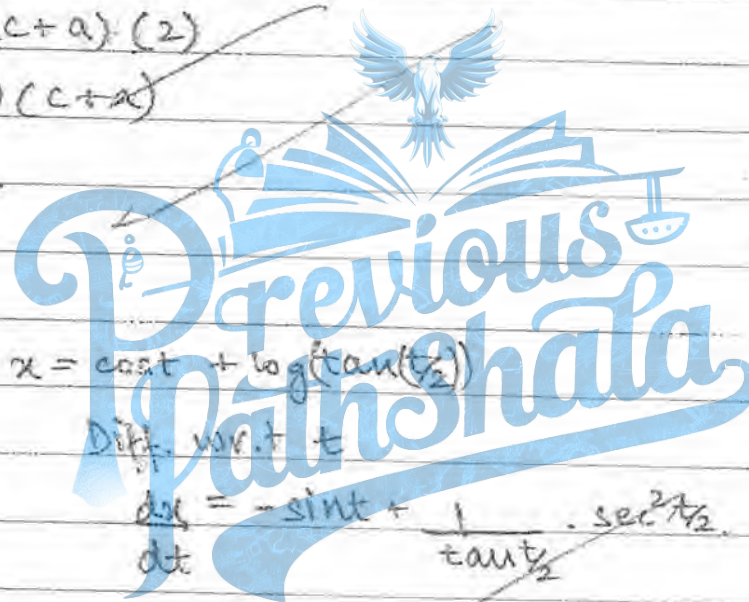
Taking (a+c)
common from c_1

Expanding along c_1

$$\Delta = (a+b)(b+c)(c+a) (2)$$

$$\Delta = 2(a+b)(b+c)(c+a)$$

Hence proved.



Q19

$$y = \sin t$$

Diff. wrt t

$$\frac{dy}{dt} = \cos t \quad \text{--- (1)}$$

Diff w.r. t

$$\frac{d^2y}{dt^2} = -\sin t$$

$$\left[\frac{d^2y}{dt^2} \right]_{t=\frac{\pi}{4}} = -\sin \frac{\pi}{4}$$

$$\left[\frac{d^2y}{dt^2} \right]_{t=\frac{\pi}{4}} = -\frac{1}{\sqrt{2}}$$

$$x = \cos t + \log(\tan \frac{t}{2})$$

Diff. wrt t

$$\frac{dx}{dt} = -\sin t + \frac{1}{\tan \frac{t}{2}} \cdot \sec^2 \frac{t}{2} \cdot \frac{1}{2}$$

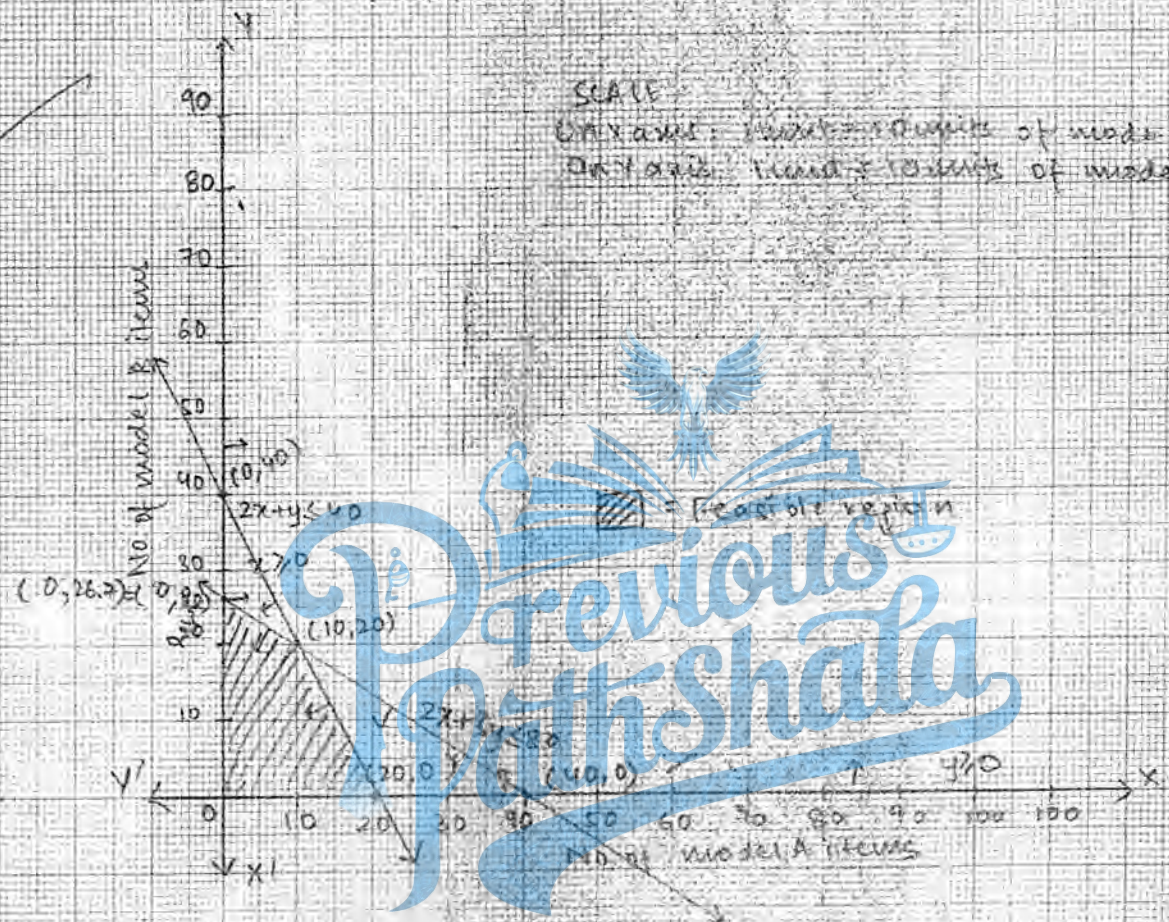
$$= -\sin t + \frac{\cos t}{2 \sin \frac{t}{2} \cos^2 \frac{t}{2}}$$

$$= -\sin t + \frac{1}{\sin t} \quad [\because 2 \sin A \cos A = \sin 2A]$$

$$\left[\frac{dx}{dt} \right]_{t=\frac{\pi}{4}} = -\sin \frac{\pi}{4} + \frac{1}{\sin \frac{\pi}{4}} \quad \frac{dx}{dt} = \frac{\cos^2 t}{\sin t}$$

P.T.D

Q 29



Q19 Contd...

Diff Divide eqn (1) by (2)

$$\frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t + \frac{1}{\sin t}}$$

$$\frac{dy}{dx} = \frac{\cos t \cdot \sin t}{1 - \sin^2 t} = \frac{\sin t}{\cos t} = \tan t$$

Diff. w.r.t x .

$$\frac{d^2y}{dx^2} = \frac{d(\tan t)}{dt} \frac{dt}{dx}$$

$$= \sec^2 t \cdot \frac{1}{dx/dt} = \frac{\sec^2 t}{-\sin t + \frac{1}{\sin t}}$$

$$= \frac{\sec^2 t \cdot \sin t}{1 - \sin^2 t} = \frac{\sin t}{\cos^2 t \cdot \cos^2 t} = \frac{\sin t}{\cos^4 t}$$

$$= \sin t \cdot \sec^4 t$$

$$\left. \frac{d^2y}{dx^2} \right|_{t=\pi/4} = \frac{1}{\sqrt{2}} (\sqrt{2})^4 = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\therefore \left[\frac{d^2y}{dx^2} \right]_{t=\pi/4} = 2\sqrt{2}$$

Q20

$$R = \{(a, b) : a \leq b\}$$

REFLEXIVE:

Every element $a \in \mathbb{R}$ is equal to itself

$$\Rightarrow a = a$$

$$\Rightarrow a \leq a \text{ is true}$$

$\therefore (a, a) \in R$ for all $a \in \mathbb{R}$ where $\mathbb{R}$ = set of all Real nos
The relation is REFLEXIVE R-Relation.

TRANSITIVE:

For all $(a, b) \in R$ and $(b, c) \in R$

$$a \leq b \text{ and } b \leq c$$

where $a, b, c \in \mathbb{R}$

$$\Rightarrow a \leq c$$

$$\Rightarrow (a, c) \in R$$

$\therefore$ The set is $\leq$ relation is TRANSITIVE

$\therefore$ for $(a, b), (b, c) \in R$, $(a, c) \in R$

Q21

SYMMETRIC : For relation to be symmetric,
for all $(a, b) \in R$, (b, a) should also exist in R .

$$a \leq b \not\Rightarrow b \leq a \quad \rightarrow \text{This relation is true only } a=b=1.$$

For eg: $\frac{1}{2} \leq 1 \Rightarrow (\frac{1}{2}, 1) \in R$

but $1 \not\leq \frac{1}{2} \therefore (1, \frac{1}{2}) \notin R$

$\therefore$ Relation is NOT SYMMETRIC

Q21

$$y = \sqrt{3x-2}$$

$$\left. \frac{dy}{dx} \right|_{(x,y)} = \frac{3}{2\sqrt{3x-2}} = \frac{3}{2y_1} = \text{slope of tangent} = m$$

Acc. to ques $m = 2$

$$\Rightarrow \frac{3}{2y_1} = 2 \Rightarrow y_1 = \frac{3}{4}$$

$\because$ Slope of $(4x-2y+5=0)$ is $m_1 = 2$.
and 11el line have equal slope

$$\Rightarrow \frac{3}{2\sqrt{3x-2}} = 2 \Rightarrow 3 = 4\sqrt{3x-2}$$

$$\Rightarrow 9 = 16(3x-2) \quad \text{squaring both side}$$

$$\Rightarrow 3x-2 = \frac{9}{16} \Rightarrow x_1 = \frac{47}{48} \Rightarrow |x_1| = \frac{47}{48}$$

$$x_1 = \frac{41}{48}$$

$$\Rightarrow y_1 = \sqrt{3x_1 - 2} = \sqrt{3 \times \frac{41}{48} - 2} = \sqrt{\frac{41-32}{16}} = \pm \frac{3}{4} = \left| \frac{3}{4} \right| = \sqrt{\frac{9}{16}}$$

At $\left(\frac{41}{48}, \frac{3}{4} \right)$ and $\left(\frac{41}{48}, -\frac{3}{4} \right)$, slope of tangent is parallel to given line.
 (But $y \neq 0 \Rightarrow y \geq 0$)

EQUATION OF TANGENT

For $x_1 = \frac{41}{48}$, $y_1 = \frac{3}{4}$

$$y - \frac{3}{4} = 2 \left(x - \frac{41}{48} \right)$$

$$y - \frac{3}{4} = 2x - \frac{41}{24}$$

$$2x - y - \frac{41}{24} + \frac{18}{24} = 0$$

$$\Rightarrow 2x - y - \frac{23}{24} = 0$$

$$48x - 24y - 23 = 0$$

For $x_1 = \frac{41}{48}$, $y_1 = -\frac{3}{4}$

$$y + \frac{3}{4} = 2 \left(x - \frac{41}{48} \right)$$

$$2x - y - \frac{41}{24} - \frac{18}{24} = 0$$

$$48x - 24y - 59 = 0$$

↑ NOT POSSIBLE

$$\text{Slope of normal} = -\frac{1}{\left(\frac{dy}{dx}\right)_{(x,y)}} = -\frac{1}{2}$$

EQUATION OF NORMAL :

At $\left(\frac{41}{48}, \frac{3}{4}\right)$

$$y - \frac{3}{4} = -\frac{1}{2}\left(x - \frac{41}{48}\right)$$

$$\Rightarrow y - \frac{3}{4} = -\frac{x}{2} + \frac{41}{96}$$

$$\Rightarrow y + \frac{x}{2} - \frac{3}{4} - \frac{41}{96} = 0$$

$$\Rightarrow y + \frac{x}{2} + \left(\frac{-72-41}{96}\right) = 0$$

$$\Rightarrow \boxed{96y + 48x - 113 = 0}$$

At $\left(\frac{41}{48}, -\frac{3}{4}\right)$

$$y + \frac{3}{4} = -\frac{1}{2}\left(x - \frac{41}{48}\right)$$

$$\Rightarrow y + \frac{3}{4} + \frac{x}{2} - \frac{41}{96} = 0$$

$$\Rightarrow y + \frac{x}{2} + \frac{72-41}{96} = 0$$

$$\Rightarrow \boxed{96y + 48x + 31 = 0}$$

NOT POSSIBLE

P.T.D

Q22 $(1+x^2) \frac{dy}{dx} + 2xy = 4x^2$

$\frac{dy}{dx} + \left(\frac{2x}{1+x^2} \right) y = \frac{4x^2}{1+x^2}$

It is linear DE of form $\frac{dy}{dx} + Py = Q$

$P = \frac{2x}{1+x^2}$

$Q = \frac{4x^2}{1+x^2}$

I.F = $e^{\int P dx} = e^{\ln(1+x^2)} = 1+x^2$

Solⁿ of DE :

$$y(1+x^2) = \int \frac{4x^2}{1+x^2} dx + C$$

$$= \int 4x^2 dx + C$$

$$y(1+x^2) = \frac{4x^3}{3} + C$$

$$x=0, y=0$$

$$\therefore c=0$$

$$\Rightarrow 3y(1+x^2) = 4x^3$$

Line I

$$\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$$

$$\Rightarrow \frac{x-1}{-3} = \frac{y-2}{\lambda/7} = \frac{z-3}{2} = \mu$$

Line II

$$\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$$

$$\frac{x-1}{-3\lambda/7} = \frac{y-5}{1} = \frac{z-6}{-5} = \mu$$

Direction ratios are $(-\frac{3\lambda}{7}, 2)$ and $(-\frac{3\lambda}{7}, 1, -5)$ respectively
 (a_1, a_2, a_3) (b_1, b_2, b_3)

For lines to be $\perp$, $\vec{a} \cdot \vec{b} = 0$ where $(\vec{a}, \vec{b})$ are $\parallel$ vectors of lines
 $a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$

$$\frac{9\lambda}{7} + \frac{\lambda}{7} - 10 = 0$$

$$\frac{10\lambda}{7} = 10$$

$$\boxed{\lambda = 7}$$

$\therefore$ For $\lambda = 7$, lines are $\perp$.

Any point of line I is $(-3\beta+1, \beta+2, 2\beta+3)$

line II is $(-3\mu+1, \mu+5, -5\mu+6)$

For lines to intersect, they should be equal.

$$-3\beta+1 = -3\mu+1$$

$$\mu+5 = \beta+2$$

$$2\beta+3 = -5\mu+6$$

$$3\mu - 3\beta = 0$$

$$\mu - \beta + 3 = 0 \quad \text{--- (3)}$$

$$2\beta + 5\mu = 3 \quad \text{--- (2)}$$

$$\mu = \beta \quad \text{--- (1)}$$

From (1), (2)

$$7\mu = 3$$

$$\boxed{\mu = \frac{3}{7}} \quad \boxed{\beta = \frac{3}{7}}$$

In eqn (3)

$$\frac{3}{7} - \frac{3}{7} + 3 \neq 0$$

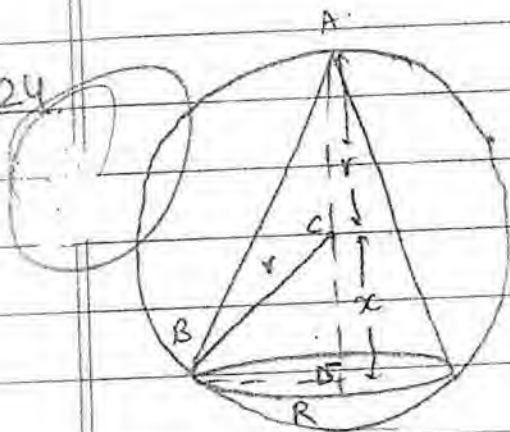
$\therefore$ Values do not satisfy eqn (3)

$\Rightarrow$ LINES do NOT intersect.

P.T.O

SECTION-D

Q24



Consider a sphere of radius r .

Then, height of cone $= r+x = H$

Radius of cone $= \sqrt{r^2 - x^2} = R$

$$\text{Volume of cone} = V = \frac{1}{3} \pi R^2 H$$

$$V = \frac{1}{3} \pi (r^2 - x^2) (r+x)$$

$$= \frac{\pi}{3} (r^3 - rx^2 + xr^2 - x^3)$$

$$\frac{dV}{dx} = \frac{\pi}{3} (-2rx + r^2 - 3x^2)$$

For critical pt $\frac{dV}{dx} = 0$

$$r^2 - 2rx - 3x^2 = 0$$

$$r^2 - 3rx + rx - 3x^2 = 0$$

$$(r-x)(r-3x) = 0$$

x cannot be $-ve$

$$\boxed{x = \frac{r}{3}} \text{ is critical pt}$$

$$\frac{d^2V}{dx^2} = \frac{\pi}{3} (-2r - 6x)$$

$$\left. \frac{d^2V}{dx^2} \right|_{x=r/3} = \frac{\pi}{3} [-2r - 2r] = -\frac{4\pi r}{3} < 0$$

Using Second $\therefore \left[\begin{array}{c} x=r \\ 3 \end{array} \right]$ is a ~~max~~ point of maxima
Derivative test,

$$H = r + x$$

$$\left[\begin{array}{c} H = \frac{4r}{3} \end{array} \right]$$

Hence proved

$$\text{Max volume of cone} = V_{\max} = \frac{\pi}{3} (r+x)(r^2-x^2)$$

$$= \frac{\pi}{3} \left(\frac{4r}{3} \right) \left(r^2 - \frac{r^2}{9} \right)$$

$$= \frac{4\pi r^3}{81} \cdot 8 = \frac{32\pi r^3}{81} \text{ (unit)}^3$$

$$\left[V_{\max} = \frac{32\pi r^3}{81} \text{ (unit)}^3 \right]$$

Q25

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 0 & +2 & 1 \\ -1 & -9 & -5 \\ 2 & +23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$|A| = 2(0) + 3(-2) + 5(1) \\ = 0 - 6 + 5 = -1$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = -1 \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Given Eqⁿ:

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

$$AX = B$$

Pre-multiply A^{-1} on both sides

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{array}{l} x \\ y \\ z \end{array} \begin{array}{l} \boxed{x=1} \\ \boxed{y=2} \\ \boxed{z=3} \end{array}$$

Q26 R.E.

Q26.

A = cho Event of choosing a defective item.

E_1 = Item produced by A

E_2 = Item produced by B

E_3 = Item produced by C

$$P(E_1) = \frac{50}{100} = \frac{1}{2}, \quad P(E_2) = \frac{20}{100} = \frac{2}{10}, \quad P(E_3) = \frac{20}{100} = \frac{1}{5}$$

$$P\left(\frac{A}{E_1}\right) = \frac{1}{100}, \quad P\left(\frac{A}{E_2}\right) = \frac{5}{100} = \frac{1}{20}, \quad P\left(\frac{A}{E_3}\right) = \frac{7}{100}$$

Using Baye's Thm.

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) P\left(\frac{A}{E_1}\right)}{P(E_1) P\left(\frac{A}{E_1}\right) + P(E_2) P\left(\frac{A}{E_2}\right) + P(E_3) P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{100}}{\frac{1}{2} \times \frac{1}{100} + \frac{2}{10} \times \frac{1}{20} + \frac{1}{5} \times \frac{7}{100}}$$

$$= \frac{\frac{1}{200}}{\frac{1}{200} + \frac{3}{200} + \frac{7}{500}} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{3}{2} + \frac{7}{5}} = \frac{\frac{1}{2}}{2 + \frac{7}{5}} = \frac{\frac{1}{2}}{\frac{17}{5}}$$

$$P\left(\frac{E_1}{A}\right) = \frac{5}{34} = \text{Probability that defective item was produced by A}$$

Q27

Let $\vec{n} = A\hat{i} + B\hat{j} + C\hat{k}$ of the required plane

Eqⁿ of plane is $A(x-2) + B(y-2) + C(z+1) = 0$ — (3)

The plane also passes through $(3, 4, 2)$ & $(7, 0, 6)$

$$A + 2B + 3C = 0 \quad \text{--- (1)}$$

$$5A - 2B + 7C = 0 \quad \text{--- (2)}$$

Adding (1), (2)

$$6A + 10C = 0$$

$$\boxed{A = -\frac{5C}{3}}$$

In eqn (2) $-\frac{25C}{3} + 7C = 2B$

$$\frac{-4C}{2 \times 3} = B$$

$$\Rightarrow \boxed{B = -\frac{2C}{3}}$$

$$\vec{n} = -\frac{5C}{3}\hat{i} - \frac{2C}{3}\hat{j} + C\hat{k} = -\frac{C}{3}(5\hat{i} + 2\hat{j} - 3\hat{k})$$

Q29.

let x be no. of A models
 y be no. of B models.

Objective $Z = 15x + 10y$ (Maximize)

Subject to constraints:

$$x, y \geq 0$$

$$2x + y \leq 40 \quad (\text{skilled man working hrs})$$

$$2x + 3y \leq 80 \quad (\text{Semi-skilled man working hrs})$$

$$2x + y \leq 40$$

$$2x + y = 40$$

$$x \quad 0 \quad 20$$

$$y \quad 40 \quad 0$$

Zero test: TRUE

$$2x + 3y \leq 80$$

$$2x + 3y = 80$$

$$x \quad 10 \quad 40$$

$$y \quad 0 \quad 26.67$$

Zero test: TRUE

GRAPH: On graph paper.

Previous Pathshala

Eqⁿ of line BC:

$$(y-2) = \frac{5}{-2} (x-6)$$

$$\Rightarrow y-2 = -\frac{5}{2}x + 15$$

$$\Rightarrow \boxed{y_2 = -\frac{5}{2}x + 17}$$

Eqⁿ of line AC

$$(y-5) = \frac{3}{-4} (x-2)$$

$$y-5 = -\frac{3}{4}x + \frac{3}{2}$$

$$\boxed{y_3 = -\frac{3}{4}x + \frac{13}{2}}$$

Area of shaded region is required area $A = \text{ar}(ABDE) + \text{ar}(BCFD) - \text{ar}(ACFE)$

$$A = \int_2^4 y_1 dx + \int_4^6 y_2 dx - \int_2^6 y_3 dx$$

$$= \int_2^4 (x+3) dx + \int_4^6 \left(-\frac{5}{2}x + 17\right) dx - \int_2^6 \left(-\frac{3}{4}x + \frac{13}{2}\right) dx$$

$$A = \left[\frac{x^2}{2} + 3x \right]_2^4 + \left[17x - \frac{5x^2}{4} \right]_4^6 - \left[\frac{13x}{2} - \frac{3x^2}{8} \right]_2^6$$

$$= [8 + 12 - 2 - 6] + [102 - 45 - 68 + 20] - [39 - \frac{27}{2} - 13 + \frac{3}{2}]$$

$$= 12 + 9 - 14$$

$$\boxed{A = 7 \text{ sq. units}}$$

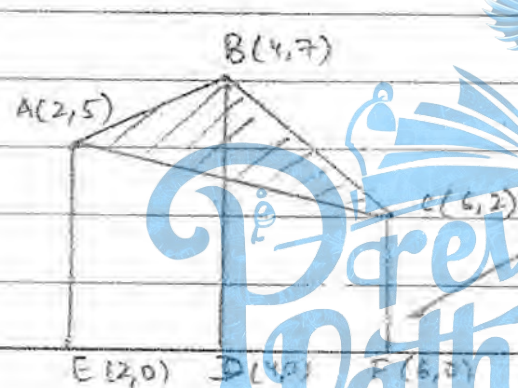
∴ Eqⁿ of other plane is

$$\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 23$$

$$5x + 2y - 3z = 23 \rightarrow \text{Cartesian eq}^n$$

$$\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) - 23 = 0$$

Q28



Eqⁿ of general line:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Eqⁿ of line AB :

$$(y - 5) = \frac{2}{2} (x - 2)$$

$$y = x + 3$$

Eqⁿ of line

Direction ratios of normal to the plane are

$$\left(-\frac{5c}{3}, -\frac{2c}{3}, c\right) \equiv (-5, -2, 3) \quad (5, 2, -3)$$

$$\therefore \vec{n} = 5\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\text{Eq}^n \text{ of plane: } 5(x-2) + 2(y-2) - 3(z+1) = 0$$

[From (3)]

$$5x + 2y - 3z - 10 - 4 - 3 = 0$$

$$\boxed{5x + 2y - 3z = 17} \text{ is cartesian eq}^n \text{ of plane.}$$

VECTOR EQN:

$$[\vec{r} \cdot \vec{n} = d] \Rightarrow [\vec{r} - (2\hat{i} + 2\hat{j} - \hat{k})] \cdot \vec{n} = 0 \quad [\because (\vec{r} - \vec{a}) \cdot \vec{n} = 0 \text{ is eq}^n \text{ of plane}]$$

$$\vec{r}(5\hat{i} + 2\hat{j} - 3\hat{k}) - (2\hat{i} + 2\hat{j} - \hat{k})(5\hat{i} + 2\hat{j} - 3\hat{k}) = 0$$

$$\Rightarrow \boxed{\vec{r}(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17} \text{ is vector eq}^n \text{ of plane.}$$

For plane parallel to above plane, $\vec{n}_2 = \vec{n} = 5\hat{i} + 2\hat{j} - 3\hat{k}$

$$(\vec{r} - \vec{a}_1) \cdot \vec{n} = 0$$

$$\text{Here } \vec{a}_1 = 4\hat{i} + 3\hat{j} + \hat{k}$$

$$[\vec{r} - (4\hat{i} + 3\hat{j} + \hat{k})] \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 0$$

$$\vec{r}(5\hat{i} + 2\hat{j} - 3\hat{k}) - (20 + 6 - 3) = 0$$

Corner pt

$$Z = 15x + 10y$$

(0,0)

$$Z = 0$$

~~(0,0)~~ (0,80)

$$Z = 400 - 0 + 800 = 200 \approx 207$$

(10,20)

$$Z = 150 + 200 = 350$$

(MAX)

(20,0)

$$Z = 300$$

 $\therefore$ No. of model A = 10

No. of model B = 20

Maximum profit = ₹ 350