

Ans 29°

6

Let x and y be the decision variables where,

x represents the packets of Screw A and

y represents the packets of Screw B.

$$Z = 70x + 100y \quad \text{to be maximised}$$

Subject to constraints,

$$4x + 6y \leq 240$$

$$6x + 3y \leq 240$$

$$x \geq 0, y \geq 0 \quad (\text{non-negative constraints})$$

Converting above inequalities into equations,

$$4x + 6y = 240$$

$$\Rightarrow 2x + 3y = 120$$

x	0	60
y	40	0

$$6x + 3y = 240$$

$$\Rightarrow 2x + y = 80$$

x	0	40
y	80	0

$x = 0$ and $y = 0$ (non-negative constraints)

Consider a test point (0,0)

$$2x + 3y \leq 120$$

$$0 + 0 \leq 120$$

$$0 < 120$$

which is true

$$2x + y \leq 80$$

$$0 + 0 \leq 80$$

$$0 < 80$$

which is true

Corner points

O (0, 0)

A (0, 40)

B (30, 20)

C (40, 0)

$$Z = 70x + 100y$$

$$Z = 0 + 0 = 0$$

$$Z = 0 + 100(40) = 4000 \rightarrow \text{maximum value}$$

$$Z = 70(30) + 100(20) = 3800$$

$$Z = 70(40) + 100(0) = 2800$$

The factory owner must produce 0 packets of screw A and 40 packets of screw B to maximize his profit.

His maximum profit = 4000 paise or Rs 40

graph
→

Corner points

O (0, 0)

A (0, 40)

B (30, 20)

C (40, 0)

$$Z = 70x + 100y$$

$$Z = 0 + 0 = 0$$

$$Z = 70(0) + 100(40) = 4000$$

$$Z = 70(30) + 100(20) = 4100 \rightarrow \text{maximum value}$$

$$Z = 70(40) + 100(0) = 2800$$

The factory owner must produce 30 packets of screw A and 20 packets of screw B to maximize his profit.

His maximum profit = 4100 paise or Rs 41

Ans 28°

Equation of line in cartesian form:-

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = \lambda$$

$$\Rightarrow x = 3\lambda + 2$$

$$y = 4\lambda - 1$$

$$z = 2\lambda + 2$$

$$\Rightarrow P(3\lambda + 2, 4\lambda - 1, 2\lambda + 2)$$

Equation of plane in cartesian form:-

$$x - y + z = 5$$

The line and plane intersect at the point of line must satisfy the equation of the plane

$$x - y + z = 5$$

$$\Rightarrow (3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) = 5$$

$$3\lambda + 2 - 4\lambda + 1 + 2\lambda + 2 = 5$$

$$\lambda + 5 = 5$$

$$\boxed{\lambda = 0}$$

$$\text{So, the required point } P(3(0) + 2, 4(0) - 1, 2(0) + 2) \\ = P(2, -1, 2)$$

Distance of the point $Q(-1, -5, -10)$ from $P(2, -1, 2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$d = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$$

$$d = \sqrt{9+16+144}$$

$$d = \sqrt{169}$$

$$d = 13 \text{ units}$$

Ans 27°

6

$$\int_0^{\pi/4}$$

$$\frac{\sin x + \cos x}{16 + 9 \sin 2x} dx$$

Put $\sin x - \cos x = t$

$$(\cos x + \sin x) dx = dt$$

Also $(\sin x - \cos x)^2 = t^2$

$$\sin^2 x + \cos^2 x - 2 \sin x \cos x = t^2$$

$$1 - \sin 2x = t^2$$

$$\sin 2x = 1 - t^2$$

limits, when, $x=0, t=-1$

$x=\pi/4, t=0$

$$\int_{-1}^0 \frac{dt}{16+9(1-t^2)}$$

$$= \int_{-1}^0 \frac{dt}{16+9-9t^2}$$

$$= \int_{-1}^0 \frac{dt}{25-9t^2}$$

$$= \int_{-1}^0 \frac{dt}{-9(t^2 - 25/9)}$$

$$= -\frac{1}{9} \int_{-1}^0 \frac{dt}{t^2 - (5/3)^2}$$

$$= -\frac{1}{9} \left[\frac{1}{2 \times \frac{5}{3}} \log \left| \frac{t-5/3}{t+5/3} \right| \right]_{-1}^0$$

$$= -\frac{1}{9} \left[\frac{3}{10} \log \left| \frac{3t-5}{3t+5} \right| \right]_{-1}^0$$

$$= -\frac{1}{9} \times \frac{3}{10} \left[\log \left| \frac{3(0)-5}{3(0)+5} \right| - \log \left| \frac{3(-1)-5}{3(-1)+5} \right| \right]$$

$$= -\frac{1}{30} \left[\log \left| \frac{-5}{5} \right| - \log \left| \frac{-8}{2} \right| \right]$$

$$= -\frac{1}{30} \left[\log |-1| - \log |-4| \right] = -\frac{1}{30} \left[\log \left| \frac{-1}{-4} \right| \right] = -\frac{1}{30} \log \left(\frac{1}{4} \right)$$

$$= \frac{1}{15} \log 2$$

Ans 26°

6

Circle,

$$x^2 + y^2 = 32 \quad \text{--- (i)}$$

centre (0,0)

$$\text{radius} = 4\sqrt{2} \text{ cm.}$$

line, $y = x$ --- (ii)

on solving (i) and (ii) eq

$$x^2 + x^2 = 32$$

$$2x^2 = 32$$

$$x^2 = 16$$

$$x = \pm 4$$

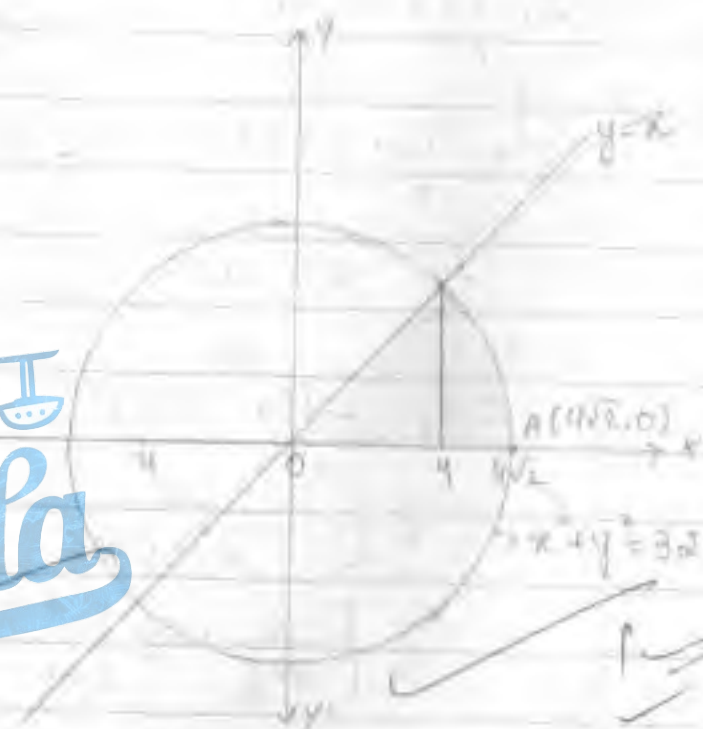
$$y = \pm 4$$

Required Area:

$$\int_0^4 x \, dx + \int_4^{4\sqrt{2}} \sqrt{32-x^2} \, dx$$

$$= \left(\frac{x^2}{2} \right)_0^4 + \left(\frac{x}{2} \sqrt{32-x^2} + \frac{32}{2} \sin^{-1} \frac{x}{4\sqrt{2}} \right)_{4\sqrt{2}}$$

$$= \left(\frac{x^2}{2} \right)_0^4 + \left(\frac{x}{2} \sqrt{32-x^2} + 16 \sin^{-1} \frac{x}{4\sqrt{2}} \right)_{4\sqrt{2}}$$



$$\frac{1}{2} + \frac{1}{2}$$

2

$$= \left(\frac{16}{2} - \frac{0}{2} \right) + \left(\frac{4\sqrt{2}}{2} \sqrt{32-32} + 16 \sin^{-1} \frac{4\sqrt{2}}{4\sqrt{2}} - \left[\frac{4}{2} \sqrt{32-16} + 16 \sin^{-1} \frac{4}{4\sqrt{2}} \right] \right)$$

$$= (8-0) + (0 + 16 \sin^{-1}(1) - 2\sqrt{16} - 16 \sin^{-1} \frac{1}{\sqrt{2}})$$

$$= 8 + (16 \times \pi/2 - 2 \times 4 - 16 \times \pi/4)$$

$$= 8 + 8\pi - 8 - 4\pi$$

$$= 4\pi \text{ units}$$

Ans 25.

(6)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

By elementary row transformation

$$A = I A$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$R_3 \rightarrow R_3 + 2R_1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

$$R_1 \rightarrow R_1 - 2R_2$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

$$R_1 \rightarrow R_1 - R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 - R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$$

$$I = A^T A$$

Therefore, $A^T = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

Ques 240

To show: R is an equivalence relation
Solution.

For Reflexive :-

$$(a, a) : a, a \in A$$

$$a R a \quad \forall a \in A$$

$|a - a|$ is divisible by 4

0 is divisible by 4

which is true

$\Rightarrow R$ is a reflexive relation.

For Symmetric relation :-

Let $(a, b) \in R \quad \forall a, b \in A$

$a R b \quad \forall a, b \in A$

$|a-b|$ is divisible by 4

$\Rightarrow |b-a|$ is divisible by 4

$\Rightarrow b R a \quad \forall b, a \in A$

$\Rightarrow (b, a) \in R$

$\Rightarrow R$ is a symmetric relation since $(a, b) \in R$ and $(b, a) \in R$

For transitive relation :-

Let $(a, b) \in R$ and $(b, c) \in R \quad \forall a, b, c \in A$

$a R b$ and $b R c \quad \forall a, b, c \in A$

$|a-b|$ is divisible by 4 and $|b-c|$ is divisible by 4

$$|a-b| = 4\lambda \quad \text{--- (i)}$$

$$|b-c| = 4\mu \quad \text{--- (ii)}$$

$$(i) + (ii)$$

$$|a-b+b-c| = 4(\lambda + \mu)$$

$$|a-c| = 4(\lambda + \mu)$$

$\Rightarrow a R c \quad \forall a, c \in A$

$$\Rightarrow (a, c) \in R$$

$\Rightarrow R$ is an transitive relation since $(a, b) \in R$, $(b, c) \in R$ and also $(a, c) \in R$

Hence R is reflexive, symmetric and transitive so it is a equivalence relation.

The set of all elements related to A are:-

$$R = \{(1, 5), (5, 1), (1, 9), (9, 1), (1, 1)\}$$

$$\text{Equivalence class } [2] = \{2, 6, 10\}$$

Ans 23.

Let X denote the larger of the dice numbers.

$$X = 2, 3, 4, 5$$

$$P(X=2) = \frac{2}{20} = \frac{1}{10}$$

$$P(X=3) = \frac{4}{20} = \frac{2}{10}$$

$$P(X=4) = \frac{6}{20} = \frac{3}{10}$$

$$P(X=5) = \frac{8}{20} = \frac{4}{10}$$

Probability Distribution :-

X	2	3	4	5
P(X)	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

Mean

$$\Sigma(X) = \Sigma P_i \cdot x_i$$

$$\Sigma P_i \cdot x_i = \frac{2 \times 1}{10} + \frac{3 \times 2}{10} + \frac{4 \times 3}{10} + \frac{5 \times 4}{10}$$

$$\Sigma P_i \cdot x_i = \frac{2}{10} + \frac{6}{10} + \frac{12}{10} + \frac{20}{10} = \frac{40}{10} = \boxed{4} \text{ ans}$$

$$\frac{1}{2} + \frac{1}{2}$$

Variance

$$\Sigma P_i (x_i^2) - (\Sigma P_i x_i)^2$$

$$\sigma = \left(\frac{4 \times 1}{10} + \frac{9 \times 2}{10} + \frac{16 \times 3}{10} + \frac{25 \times 4}{10} \right) - (16)$$

$$= \left(\frac{4}{10} + \frac{18}{10} + \frac{48}{10} + \frac{100}{10} \right) - (16)$$

$$= \left(\frac{170}{10} \right) - 16$$

$$= 17 - 16 = \boxed{1} \text{ ans}$$

$$\frac{1}{2} + \frac{1}{2}$$

Standard deviation $\sqrt{1} = 1 \text{ ans}$

Ques 22. Let E_1 = "Event that the girl threw 3, 4, 5 or 6"
 E_2 = "Event that the girl threw 1, 2"
 and A = "Event that the girl got exactly one tail"

$$P(E_1) = \frac{4}{6} = \frac{2}{3}, \quad P(E_2) = \frac{2}{6} = \frac{1}{3}$$

$$P(A/E_1) = \frac{1}{2}, \quad P(A/E_2) = \frac{3}{8}$$

$$\text{Now, } P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

$$P(E_1/A) = \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{3}{8}}$$

$$P(E_1/A) = \frac{\frac{2}{6}}{\frac{2}{6} + \frac{1}{8}} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{8}} = \frac{\frac{8+3}{24}}{\frac{11}{24}}$$

$$P(E_1/A) = \frac{1}{3} \times \frac{24}{11} = \boxed{\frac{8}{11}} \text{ ans.}$$

Ans 21:

(4)

$$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$$

$$\vec{a}_1 = 4\hat{i} - \hat{j} + 0\hat{k}$$

$$\vec{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$$

$$\vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{b}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}$$

$$\text{Shortest distance} = \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}$$

Now,

$$\begin{aligned} \vec{a}_2 - \vec{a}_1 &= (\hat{i} - \hat{j} + 2\hat{k}) - (4\hat{i} - \hat{j} + 0\hat{k}) \\ &= -3\hat{i} + 0\hat{j} + 2\hat{k} \\ &= -3\hat{i} + 2\hat{k} \end{aligned}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$$

$$b_1 \vec{r} \times b_2 \vec{r} = \hat{i}(-10+12) - \hat{j}(-5+6) + \hat{k}(4-4)$$

$$|b_1 \vec{r} \times b_2 \vec{r}| = \sqrt{4+1+0} = \sqrt{5}$$

$$b_1 \vec{r} \times b_2 \vec{r} = 2\hat{i} - \hat{j} + 0\hat{k}$$

Now,

$$d = \frac{|(-3\hat{i} + 0\hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j} + 0\hat{k})|}{\sqrt{4+1+0}}$$

$$d = \frac{|-3 \times 2 + 0 \times -1 + 2 \times 0|}{\sqrt{5}}$$

$$d = \frac{|-6+0+0|}{\sqrt{5}}$$

$$d = \frac{6}{\sqrt{5}} \text{ units}$$

Ans 20. Let $\vec{d}$ be $x\hat{i} + y\hat{j} + z\hat{k}$

Now, $\vec{d} \perp \vec{c}$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} + \hat{j} - \hat{k}) = 0$$

$$3x + y - z = 0 \quad \text{--- (i)}$$

Also, $\vec{d} \perp \vec{b}$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} - 4\hat{j} + 5\hat{k}) = 0$$

$$x - 4y + 5z = 0 \quad \text{--- (ii)}$$

Also, $\vec{d} \cdot \vec{a} = 21$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (4\hat{i} + 5\hat{j} - \hat{k}) = 21$$

$$4x + 5y - z = 21 \quad \text{--- (iii)}$$

Solving (i) and (iii) equations

$$3x + y - z = 0$$

$$-4x + 5y - z = 21$$

$$-x - 4y = -21$$

$$\rightarrow x + 4y = 21 \quad \text{--- (iv)}$$

Multiply (iv) by 5 and add to (iii) eq.

$$15x + 20y - 5z = 0$$

$$+ x + 4y + 5z = 0$$

$$16x + 24y = 0 \quad \text{--- (v)}$$

Solving (iv) and (v) eq.

$$x + 4y = 21$$

$$-64x + 24y = 0$$

$$-63x = 21$$

$$x = \frac{-21}{63} = -\frac{7}{21} = -\frac{1}{3}$$

Putting value of x in eq. (v).

$$x + 4y = 21$$

$$-\frac{1}{3} + 4y = 21$$

$$4y = 21 + \frac{1}{3}$$

$$4y = \frac{63+1}{3}$$

$$4y = \frac{64}{3}$$

$$y = \frac{64}{12} = \frac{32}{6} = \frac{16}{3}$$

Also,

$$3x + y - z = 0$$

$$3\left(-\frac{1}{3}\right) + \frac{16}{3} = z$$

$$-1 + \frac{16}{3} = z$$

$$-\frac{3+16}{3} = z, \quad z = \frac{13}{3}$$

Therefore, $\vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}$ Ans

$$= \frac{1}{3}(-\hat{i} + 16\hat{j} + 13\hat{k})$$
 Ans

Ques 19.

4

$$\frac{dy}{dx} + 2y \tan x = \sin x$$

On comparing the above equation with the standard linear equation

$$\frac{dy}{dx} + Py = Q$$

we get, $P = 2 \tan x$, $Q = \sin x$

therefore, I.F. = $e^{\int P dx}$

$$\begin{aligned} \text{I.F.} &= e^{\int 2 \tan x dx} \\ &= e^{2(\log \sec x)} \\ &= e^{\log(\sec x)^2} = \sec^2 x \end{aligned}$$

Now,

$$y \cdot \text{I.F.} = \int Q \times \text{I.F.} dx$$

$$y \cdot \sec^2 x = \int \sin x \times \sec^2 x dx$$

$$y \cdot \sec^2 x = \int \sin x \times \frac{1}{\cos^2 x} dx$$

$$y \cdot \sec^2 x = \int \tan x \cdot \sec x dx$$

Put $\sec x = t$

$$(\sec x \tan x) dx = dt$$

$$y \cdot \sec^2 x = \int 1 \, dx$$

$$y \cdot \sec^2 x = x + C$$

$$y \cdot \sec^2 x = \sec x + C$$

Now when $y = 0$, $x = \pi/3$.

$$0 = \sec \frac{\pi}{3} + C$$



$$0 = 1 + C \Rightarrow C = -1$$

$$C = -2$$

Therefore,

Solution =

$$y \cdot \sec^2 x = \sec x - 2$$

$$\text{or } y = \frac{1}{\sec x} - \frac{2}{\sec^2 x}$$

$$\text{or } y = \sec x^{-1} - 2(\sec^2 x)^{-1} \quad \underline{\text{Ans}}$$

$$+ \frac{1}{2}$$

$$\frac{1}{2}$$

$$\neq$$

Ans 18.

(4)

$$\int \frac{2 \cos x}{(1 - \sin x)(1 + \sin^2 x)} dx$$

$$\text{Put } \sin x = t$$

$$\cos x dx = dt$$

$$2 \int \frac{dt}{(1-t)(1+t^2)}$$

Now by partial fraction,

$$\frac{1}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2}$$

$$1 = A(1+t^2) + (Bt+C)(1-t)$$

$$1 = A + At^2 + Bt - Bt^2 + C - Ct$$

$$A + C = 1 \quad \text{--- (i)}$$

$$A - B = 0 \quad \text{--- (ii)}$$

$$B - C = 0 \quad \text{--- (iii)}$$

$$(i) + (iii)$$

$$A + C + B - C = 1$$

$$A + B = 1 \quad \text{--- (iv)}$$

$$A - B = 0 \quad \text{--- (ii)}$$

$$2A = 1$$

$$A = 1/2, \quad B = 1/2, \quad C = 1/2$$

Ans 99.

Scale
 x -axis : 1 cm = 10 units
 y -axis : 1 cm = 10 units



$$2 \int \frac{1/2}{1-t} + \frac{1/2 t + 1/2}{1+t^2} dt$$

$$2 \times \frac{1}{2} \int \frac{dt}{1-t} + 2 \times \frac{1}{2} \int \frac{t+1}{t^2+1} dt$$

$$\int \frac{dt}{1-t} + \int \frac{t}{t^2+1} dt + \int \frac{dt}{t^2+1}$$

$$\int \frac{dt}{1-t} + \frac{1}{2} \int \frac{2t}{t^2+1} dt + \int \frac{dt}{t^2+1}$$

$$-\log(1-t) + \frac{1}{2} \log(t^2+1) + \tan^{-1} t + C.$$

$$-\log(1-\sin x) + \frac{1}{2} \log(1+\sin^2 x) + \tan^{-1}(\sin x) + C.$$

$$\Rightarrow \frac{1}{2} \log(1+\sin^2 x) - \log(1-\sin x) + \tan^{-1}(\sin x) + C \quad \underline{\text{Ans.}}$$

$$= \log(1+\sin^2 x)^{1/2} - \log(1-\sin x) + \tan^{-1}(\sin x) + C$$

$$= \log \left| \frac{\sqrt{1+\sin^2 x}}{1-\sin x} \right| + \tan^{-1}(\sin x) + C.$$

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04/2403

Ques 17°

(A) ✓

Let, length = breadth = x height = y Volume = $x \times x \times y$

$$K = x^2 y$$

$$y = \frac{K}{x^2}$$

According to question,

$$S = x^2 + 4xy$$

$$S = x^2 + 4x \times \frac{K}{x^2} = x^2 + \frac{4K}{x}$$

$$\frac{dS}{dx} = 2x - \frac{4K}{x^2} = \frac{2x^3 - 4K}{x^2}$$

Put $\frac{dS}{dx} = 0$ for critical points

$$\frac{2x^3 - 4K}{x^2} = 0$$

$$2x^3 - 4K = 0$$

$$x^3 = 2K$$

$$x = (2K)^{1/3}$$

$$\text{Now, } \frac{d^2s}{dx^2} = 2 - 4K \frac{(-2)}{x^3} = 2 + \frac{8K}{x^3}$$

$$\left(\frac{d^2s}{dx^2} \right)_{(x=(2K)^{1/3})} = \frac{2 + \frac{8K}{2K}}{2K} = 2 + 4 = 6 \quad \checkmark \quad \frac{1}{2} \quad \checkmark$$

$\frac{d^2s}{dx^2} > 0$ hence it is minimum. $\checkmark \quad \neq \frac{1}{2} \quad \checkmark$

Now,

$$y = \frac{K}{x^2}$$

$$y = \frac{K}{(2K)^{2/3}} = \frac{K \cdot K^{-2/3}}{(2)^{2/3}} = \frac{K^{1/3}}{2^{2/3}}$$

$$y = \frac{1}{2} \left(\frac{K^{1/3}}{2^{1/3}} \right)$$

$$y = \frac{1}{2} (2K)^{1/3}$$

$$\boxed{y = \frac{1}{2} x}$$

$\checkmark \quad \frac{1}{2}$ Hence Proved.

Value :-

Helping in nature

Support to middle class people

cooperative & concern towards poor.

Ques 16°

(A)

$$f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$$

$$f'(x) = \frac{4x^3}{4} - 3x^2 - 10x + 24$$

$$f'(x) = x^3 - 3x^2 - 10x + 24$$

$$f'(x) = (x-2)(x^2 - x - 12)$$

$$f'(x) = (x-2)(x^2 - 4x + 3x + 12)$$

$$f'(x) = (x-2)[x(x-4) + 3(x-4)]$$

$$f'(x) = (x-2)(x+3)(x-4)$$

$$\text{Put } f'(x) = 0$$

$$(x-2)(x+3)(x-4) = 0$$

$$x = 2, -3, 4$$

$$\begin{array}{ccccccc} & - & + & - & + & & \\ & -\infty & -3 & 2 & 4 & \infty & \end{array}$$

$f'(x) > 0$ strictly increasing
 $f'(x) < 0$ strictly decreasing

Intervals	$f'(x)$ sign	nature
$(-\infty, -3)$	-ve	decreasing
$(-3, 2)$	+ve	increasing
$(2, 4)$	-ve	decreasing
$(4, \infty)$	+ve	increasing

- (a) Strictly increasing = $(-3, 2) \cup (4, \infty)$ ✓
 (b) Strictly decreasing = $(-\infty, -3) \cup (2, 4)$ ✓

Ques 15 ✓

(A) ✓

$$y = \sin x (\sin x)$$

$$\frac{dy}{dx} = \cos(\sin x) \cdot \cos x$$

$$\cot x = \cot(\sin x) = \frac{1}{\cos x} \cdot \frac{dy}{dx}$$

$$\text{Now, } \frac{d^2y}{dx^2} = -\cos(\sin x) \sin x + \cos x (-\sin(\sin x) \cos x)$$

$$\frac{d^2y}{dx^2} = -\sin x \cos(\sin x) - \cos^2 x \sin(\sin x)$$

$$\frac{d^2y}{dx^2} = -\sin x \times \frac{1}{\cos x} \times \frac{dy}{dx} - \cos^2 x y$$

$$\frac{d^2y}{dx^2} = -\tan x \frac{dy}{dx} - \cos^2 x y$$

$$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$$

Hence Proved ✓

Ans 14.



$$x = a(2\theta - \sin 2\theta)$$

$$\frac{dx}{d\theta} = a(2 - 2\cos 2\theta)$$

$$\frac{dx}{d\theta} = 2a(1 - \cos 2\theta)$$

$$y = a(1 - \cos 2\theta)$$

$$\frac{dy}{d\theta} = a(0 + 2\sin 2\theta)$$

$$\frac{dy}{d\theta} = 2a\sin 2\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta} \times \frac{d\theta}{dx}}{\frac{dx}{d\theta}} = \frac{2a\sin 2\theta}{2a(1 - \cos 2\theta)} = \frac{2\sin 2\theta \cos \theta}{2\sin^2 \theta}$$

$$\frac{dy}{dx} = \cot \theta$$

$$\left(\frac{dy}{dx}\right)_{(\theta=\pi/3)} = \cot \frac{\pi}{3}$$

$$= \cot 60^\circ$$

$$= \boxed{\frac{1}{\sqrt{3}}} \text{ Ans}$$

Ans 13.

④ ✓

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}$$

$$C_1 \rightarrow C_1 - C_2$$

$$\begin{vmatrix} 0 & 1 & 1+3x \\ 3y & 1 & 1 \\ -3z & 1+3z & 1 \end{vmatrix}$$

taking 3 common from C₁

$$3 \begin{vmatrix} 0 & 1 & 1+3x \\ y & 1 & 1 \\ -z & 1+3z & 1 \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$3 \begin{vmatrix} 0 & 1 & -1+3x \\ y & 0 & -3x \\ -z & 1+3z & 1 \end{vmatrix}$$

Expanding along R₁

$$3 [-1(y - 3xz) + (1+3x)(y + 3xy)]$$

$$3 [-y + 3xz + y + 3xy + 3xy + 9xyz]$$

$$3 [9xyz + 3xz + 3xy + 3xy]$$

$$9(3xyz + xy + xz + zx)$$

Ans 12.

(9)

let $E_1 =$ "Event of obtaining the sum 8"

$$E_1 = \{(2,6)(6,2)(3,5)(5,3)(4,4)\}$$

and $F =$ "Event that red die result is a number less than 4."

$$F = \left\{ \begin{array}{l} (1,1)(2,1)(3,1)(4,1)(5,1)(6,1) \\ (1,2)(2,2)(3,2)(4,2)(5,2)(6,2) \\ (1,3)(2,3)(3,3)(4,3)(5,3)(6,3) \end{array} \right\}$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} \text{ or } \frac{n(E \cap F)}{n(F)}$$

$$E \cap F = \{(6,2)(5,3)\}$$

$$n(E \cap F) = \frac{2}{36}, \quad n(F) = \frac{18}{36}$$

$$P(E/F) = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \boxed{\frac{1}{9}} \text{ Ans}$$

Ans 11. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

$$|\vec{a}| = \sqrt{1+4+9} = \sqrt{14}$$

$$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$$

$$|\vec{b}| = \sqrt{9+4+1} = \sqrt{14}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -2 & 1 \end{vmatrix}$$

$$= 4\hat{i} + 8\hat{j} + 4\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{16+64+16} = \sqrt{96} = 4\sqrt{6}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$$

$$4\sqrt{6} = \sqrt{14} \times \sqrt{14} \sin \theta$$

$$4\sqrt{6} = 14 \sin \theta$$

$$\sin \theta = \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7}$$

$$\theta = \sin^{-1} \left(\frac{2\sqrt{6}}{7} \right) \text{ Ans.}$$

Ans 10.

$$y = a e^{bx+5}$$

$$\frac{dy}{dx} = a e^{bx+5} (b)$$

$$\frac{dy}{dx} = ab e^{bx+5}$$

$$\frac{dy}{dx} = by$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = b$$

Differentiating again w.r. to x

$$y \frac{d^2y}{dx^2} - \frac{dy}{dx} \times \frac{dy}{dx} = 0$$

$$\therefore y \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0 \quad \text{Ans}$$

Ans 9.

(2)

$$\int \frac{\cos^2 x + 3 \sin^2 x}{\cos^2 x} dx$$

$$\int \frac{1 - 2 \sin^2 x + 3 \sin^2 x}{\cos^2 x} dx$$

$$\int \frac{dx}{\cos^2 x}$$

$$\int \sec^2 x dx$$

$$\Rightarrow \tan x + c \quad \text{Ans}$$

Ans 8°

Q2

$$C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$$

$$C'(x) = 0.005(3x^2) - 0.02(2x) + 30$$

$$C'(x) = 0.015x^2 - 0.04x + 30$$

when $x = 3$.

$$C'(3) = 0.015(3)^2 - 0.04(3) + 30$$

$$= 0.015(9) - 0.04(3) + 30$$

$$= 0.135 - 0.12 + 30$$

$$= 30.015$$

Ans 7°

Q2

$$\tan^{-1} \left(\frac{1 + \cos x}{\sin x} \right)$$

$$y = \tan^{-1} \left(\frac{2 \cos^2 x/2}{2 \sin x/2 \cos x/2} \right)$$

$$y = \tan^{-1} \left(\frac{\cos x/2}{\sin x/2} \right)$$

$$y = \tan^{-1} (\cot x/2)$$

$$y = \tan^{-1} [\tan(\pi/2 - x/2)]$$

$$y = \frac{\pi - x}{2}$$

$$y = \frac{\pi - x}{2}$$

Differentiating w.r.t. x

$$\frac{dy}{dx} = 0 - \frac{1}{2}$$

$$\frac{dy}{dx} = -\frac{1}{2}$$

$$\frac{1}{2} \checkmark$$

Ans 6°

$$A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$|A| = 14 - 12 = 2$$

$|A| \neq 0$ hence inverse exists

$$\text{Now, } C_{11} = 7, \quad C_{12} = +4$$

$$C_{21} = +3, \quad C_{22} = 2$$

$$\text{Adj}^o(A) = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}^o(A) = \frac{1}{2} \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \quad \underline{\text{Ans}} :$$

$$\frac{1}{2} \checkmark$$

$$2A^{-1} = 9I - A$$

$$2 \times \frac{1}{2} \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = 9 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

Hence Proved.

Ans 5.



$$3 \sin^{-1} x = \sin^{-1} (3x - 4x^3)$$

$$\text{R.H.S. } \sin^{-1} (3x - 4x^3)$$

$$\text{Put } x = \sin \theta$$

$$\sin^{-1} (3 \sin \theta - 4 \sin^3 \theta)$$

$$\sin^{-1} (\sin 3\theta)$$

$$3\theta$$

$$3 \sin^{-1} x$$

$$\text{R.H.S.} = \text{L.H.S.}$$

Hence Proved

$$-\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$-\frac{1}{2} \leq \sin \theta \leq \frac{1}{2}$$

$$\sin^{-1} \left(-\frac{1}{2} \right) \leq \theta \leq \sin^{-1} \left(\frac{1}{2} \right)$$

$$-\frac{\pi}{3} \leq \theta \leq \frac{\pi}{3}$$

$$\left[-\frac{\pi}{3}, \frac{\pi}{3} \right] \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

Ans 4:

$$a \circ b = (a * b) + 3$$

$$5 \circ 10 = (5 * 10) + 3$$

$$= 10 + 3$$

$$= 13 \quad \underline{\text{Ans}}$$

Ans 3:

$$|\vec{a}| = |\vec{b}|$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\frac{9}{2} = |\vec{a}| |\vec{a}| \cos 60^\circ$$

$$\frac{9}{2} = |\vec{a}|^2 \times \frac{1}{2}$$

$$9 = |\vec{a}|^2$$

$$|\vec{a}| = 3$$

$$\text{Ans } |\vec{a}| = |\vec{b}|$$

$$|\vec{b}| = 3$$

$$|\vec{a}| = |\vec{b}| = 3 \quad \underline{\text{Ans}}$$

Ques 2:

(1)

A is a skew symmetric matrix

$$\Rightarrow A' = -A$$

$$A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & 2 & b \\ a & 0 & -1 \\ -3 & -1 & 0 \end{bmatrix}$$

$$A' = -A$$

Now,

$$\begin{bmatrix} 0 & 2 & b \\ a & 0 & -1 \\ -3 & -1 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & b \\ a & 0 & -1 \\ -3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & 3 \\ -2 & 0 & 1 \\ -b & -1 & 0 \end{bmatrix}$$

On comparing we get,

$$\boxed{\begin{matrix} b = 3 \\ a = -2 \end{matrix}}$$

Ans

Ans 1:
 (1) ✓

$$\tan^{-1} \sqrt{3} - \cot^{-1}(-\sqrt{3})$$

$$\tan^{-1} \sqrt{3} - (\pi - \cot^{-1} \sqrt{3})$$

$$\tan^{-1} \sqrt{3} - \pi + \cot^{-1} \sqrt{3}$$

$$\tan^{-1} \sqrt{3} + \cot^{-1} \sqrt{3} - \pi$$

$$\frac{\pi}{2} - \pi$$

$$\frac{\pi - 2\pi}{2} = -\frac{\pi}{2}$$

$$= -\frac{\pi}{2}$$

As we know that

$$\tan^{-1} x + \cot^{-1} x = \pi/2$$

Previous Pathshala

One Hundred only
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