

WITH GRAPH PAPER
 केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली
 सीनियर स्कूल सर्टिफिकेट परीक्षा (नका बारहवीं)
 परीक्षार्थी प्रवेश-पत्र के अनुसार भरें

विषय Subject: **MATHEMATICS**
 विषय का Subject Code: **041**
 परीक्षा का दिन एवं तिथि
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 Medium of answering the paper: **ENGLISH**

पत्रा संख्या के अन्त में लिखें
 कोड की संख्या
 Write code no. as written on
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 ① ② ③ ④

अतिरिक्त उत्तर-पुस्तिका (यदि की संख्या)
 No. of supplementary answer sheet(s) used: **NIL**

विकलांग व्यक्ति
 Person with Disabilities: **No**

किसी शारीरिक अवस्था से प्रभावित होने से संबंधित वर्ग में ☒ का चिह्न लगाएं।
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B D H S C A

A = अक्षीय दृष्टि, B = अक्षीय दृष्टि, C = अक्षीय दृष्टि, D = अक्षीय दृष्टि, H = अक्षीय दृष्टि, S = अक्षीय दृष्टि

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क्या लेखक - लिखित उत्तरों को अंकित करेंगे? **No**
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क्या पुस्तिका में (1) अक्षरों की संख्या है
 Is Number of characters given in the paper: **No**

पत्रा संख्या में 24 अक्षर हैं। इनमें से प्रत्येक अक्षर को 24 अक्षरों में लिखा जा सकता है। यदि 24 अक्षरों में से 24 अक्षरों को लिखेंगे, तो प्रत्येक अक्षर को 24 अक्षरों में लिखा जा सकता है।
 Each letter has written in one box and one has to fill blank between each part of the name. In each Candidate's Name exactly 24 letters, write first 24 letters.

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Section-A

$$\vec{r} \cdot \hat{n} = d$$

$\hat{n}$ - Unit vector perpendicular to the plane

d - Distance of the plane from origin.

$$\vec{r} \cdot (2\hat{i} + 3\hat{j} + 6\hat{k}) = 4$$

$$\vec{r} \cdot \frac{(2\hat{i} + 3\hat{j} + 6\hat{k})}{\sqrt{2^2 + 3^2 + 6^2}} = \frac{4}{\sqrt{2^2 + 3^2 + 6^2}} \Rightarrow \vec{r} \cdot \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = \frac{4}{7} \quad (\text{Distance of plane from origin} = \frac{4}{7})$$

$$\vec{r} \cdot (6\hat{i} - 9\hat{j} + 18\hat{k}) = -30$$

$$\vec{r} \cdot \frac{(6\hat{i} - 9\hat{j} + 18\hat{k})}{\sqrt{6^2 + 9^2 + 18^2}} = \frac{-30}{\sqrt{6^2 + 9^2 + 18^2}} \Rightarrow \vec{r} \cdot \left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = \frac{-10}{7} \quad (\text{Distance of plane from origin} = \frac{10}{7} \text{ in the direction opposite to the unit vector})$$

$$\therefore \text{Distance between the planes} = \frac{4}{7} - \left(\frac{-10}{7}\right) = \frac{14}{7} = \underline{\underline{2 \text{ units}}}$$

2.

$\vec{a} - \sqrt{2}\vec{b}$ is a unit vector

$$|\vec{a} - \sqrt{2}\vec{b}| = 1$$

$$|\vec{a} - \sqrt{2}\vec{b}|^2 = 1$$

$$|\vec{a}|^2 + |\sqrt{2}\vec{b}|^2 - 2\sqrt{2}\vec{a} \cdot \vec{b} = 1$$

$$1 + 2 - 2\sqrt{2}\vec{a} \cdot \vec{b} = 1$$

$$(|\vec{a}| = |\vec{b}| = 1)$$

$$\Rightarrow |\vec{a}| |\vec{b}| \cos \theta = \frac{1}{\sqrt{2}} \quad (\theta = \text{Angle between vectors } \vec{a} \text{ \& } \vec{b})$$

$$\vec{a} \cdot \vec{b} = \frac{1}{\sqrt{2}} \Rightarrow |\vec{a}| |\vec{b}| \cos \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = 45^\circ$$

(Angle between $\vec{a}$ & $\vec{b}$ is 45°)

$$3. \quad |\vec{a}| = \frac{1}{2} \quad |\vec{b}| = \frac{4}{\sqrt{3}} \quad |\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}} \quad |\vec{a} \cdot \vec{b}| = ?$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta \quad (\theta: \text{Angle between vectors } \vec{a} \text{ \& } \vec{b})$$

$$\frac{1}{\sqrt{3}} = \frac{1}{2} \times \frac{4}{\sqrt{3}} \sin \theta$$

$$\sin \theta = \frac{1}{2} \quad \theta = 30^\circ$$

$$|\vec{a} \cdot \vec{b}| = |\vec{a}| |\vec{b}| \cos \theta$$

$$= \frac{1}{2} \times \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 1$$

$$|\vec{a} \cdot \vec{b}| = 1$$

$$4. \quad A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$$

$$kA = \begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix}$$

$$\text{But given } kA = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix} = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$$

Equating individual terms,

$$2k = -8 \quad \underline{k = -4}$$

$$3k = 4a$$

$$3 \times (-4) = 4a$$

$$a = \underline{\underline{-3}}$$

$$-5k = 5b$$

$$b = -k = \underline{\underline{4}}$$

5.

$$|AB| = |A||B| \quad (\text{Provided } A \text{ \& } B \text{ are square matrices})$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = 1 \times (-1) - 2 \times 3 = -7$$

$$|B| = \begin{vmatrix} 1 & -4 \\ 3 & -2 \end{vmatrix} = 1 \times (-2) - 3 \times (-4) = 10$$

$$\therefore |AB| = -7 \times 10 = \underline{\underline{-70}}$$

6.

$$|A| = 5$$

$$|AA^T| = |A||A^T|$$

(As A is a square matrix)

$$= |A||A|$$

(As $|A| = |A^T|$)

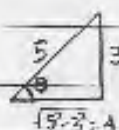
$$= |A|^2 = \underline{\underline{25}}$$

Section-B

7. To prove: $2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$

Proof: let $\sin^{-1}\left(\frac{3}{5}\right) = \theta$

$\therefore \sin \theta = \frac{3}{5}$



Triangle ①

$\therefore \tan \theta = \frac{3}{4}$ (From triangle ①)

$\therefore \theta = \tan^{-1}\left(\frac{3}{4}\right)$

Now $2\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{2 \times \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2}\right)$

$\left\{ 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) \right\}$

$= \tan^{-1}\left(\frac{\frac{3}{2}}{1 - \frac{9}{16}}\right) = \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right) = \tan^{-1}\left(\frac{3 \times 8}{7}\right) = \tan^{-1}\left(\frac{24}{7}\right)$

$\therefore 2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right)$

$= \tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24 \times 17}{7 \times 31}}\right)$

$\left\{ \tan^{-1}a - \tan^{-1}b = \tan^{-1}\left(\frac{a-b}{1+ab}\right) \right\}$

$= \tan^{-1}\left(\frac{24 \times 31 - 17 \times 7}{7 \times 31 + 24 \times 17}\right) = \tan^{-1}\left(\frac{625}{625}\right) = \tan^{-1}1 = \frac{\pi}{4}$

Hence Proved

2. Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$ ①

$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2(\frac{\pi}{2} - x)}{\sin(\frac{\pi}{2} - x) + \cos(\frac{\pi}{2} - x)} dx$

$I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin x + \cos x} dx$ ②

$$\left\{ \int_0^a f(x) dx = \int_0^a f(a-x) dx \right\}$$

Adding ① & ②

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} dx$$

$$\int_0^{\frac{\pi}{2}} \frac{dx}{\sin x + \cos x} \quad \left\{ \sin^2 x + \cos^2 x = 1 \right\} \quad \left\{ \cos(A-B) = \cos A \cos B + \sin A \sin B \right\}$$

$$I = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{dx}{\sin x + \cos x} = \frac{1}{2\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{dx}{\cos x \cdot \frac{1}{\sqrt{2}} + \sin x \cdot \frac{1}{\sqrt{2}}} = \frac{1}{2\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{dx}{\cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4}}$$

$$= \frac{1}{2\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{dx}{\cos(x - \frac{\pi}{4})}$$

Put $x - \frac{\pi}{4} = t$
 $dx = dt$

For $x=0$, $t = -\frac{\pi}{4}$, for $x = \frac{\pi}{2}$, $t = \frac{\pi}{4}$

$$\begin{aligned}
 I &= \frac{1}{2\sqrt{2}} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec t \, dt \\
 &= \frac{1}{2\sqrt{2}} \left[\log \left| \sec t + \tan t \right| \right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \\
 &= \frac{1}{2\sqrt{2}} \left(\log \left| \sec \frac{\pi}{4} + \tan \frac{\pi}{4} \right| - \log \left| \sec \left(-\frac{\pi}{4} \right) + \tan \left(-\frac{\pi}{4} \right) \right| \right) \\
 &= \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2}+1}{\sqrt{2}-1} \right| \\
 &= \frac{1}{2\sqrt{2}} \log \left| \frac{(\sqrt{2}+1)^2}{2-1} \right| = \frac{1}{\sqrt{2}} \log |\sqrt{2}+1|
 \end{aligned}$$

9. Let $I = \int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx$

$$= \int \log(\log x) \, dx + \int \frac{dx}{(\log x)^2}$$

$$= I_1 + I_2 + C \text{ (say)}$$

$\{ C = \text{Arbitrary constant} \}$

$$I_1 = \int \log(\log x) \, dx \quad \& \quad I_2 = \int \frac{dx}{(\log x)^2}$$

Consider $I_1 = \int \log(\log x) \cdot 1 \, dx$

$$= x \log(\log x) - \int x \cdot \frac{1}{\log x} \cdot \frac{1}{x} \, dx \quad \left\{ \text{Applying Integration by parts} \right\}$$

$$= x \log(\log x) - \int \frac{dx}{\log x}$$

$$= x \log(\log x) - \left[\frac{x}{\log x} - \int x \cdot \frac{-1}{(\log x)^2} \cdot \frac{1}{x} \, dx \right] \quad \left\{ \text{Applying Integration by parts} \right\}$$

$$I_1 = x \log(\log x) - \frac{x}{\log x} - \int \frac{dx}{(\log x)^2}$$

But $\int \frac{dx}{(\log x)^2} = I_2$

$$\therefore I = I_1 + I_2 + C = x \log(\log x) - \frac{x}{\log x} + C$$

$$\text{10. let } I = \int \frac{(1 - \sin x) dx}{\sin x (1 + \sin x)}$$

$$= \int \frac{(1 - \sin x)(1 - \sin x) dx}{\sin x (1 + \sin x)(1 - \sin x)}$$

{ Multiplying numerator & Denominator by $1 - \sin x$ }

$$= \int \frac{(1 - \sin x)^2 dx}{\sin x (1 - \sin^2 x)} = \int \frac{(1 + \sin^2 x - 2 \sin x) dx}{\sin x \cos^2 x}$$

$$= \int \frac{dx}{\sin x \cos^2 x} + \int \frac{\sin x dx}{\cos^2 x} = 2 \int \frac{dx}{\cos^2 x}$$

$$= \int \frac{\sin x dx}{(1 - \cos^2 x) \cos^2 x} + \int \frac{\sin x dx}{\cos^2 x} = 2 \tan x + C \quad \left\{ \int \sec^2 x \right.$$

$\underbrace{\int \frac{\sin x dx}{(1 - \cos^2 x) \cos^2 x}}_{I_1} \quad \underbrace{\int \frac{\sin x dx}{\cos^2 x}}_{I_2} \text{ (say)}$

$$I_2 = \int \frac{\sin x dx}{\cos^2 x}$$

$$\text{Put } \cos x = t$$

$$-\sin x dx = dt$$

$$\therefore I_2 = \int \frac{-dt}{t^2} = \frac{1}{t} + C_1 = \sec x + C_1$$

$$I_1 = \int \frac{\sin x \, dx}{(1 - \cos^2 x)(\cos^2 x)}$$

$$\text{Put } \cos x = u$$

$$-\sin x \, dx = du$$

$$I_1 = \int \frac{-du}{(1-u^2)u^2} = \int \frac{du}{(u^2-1)u^2} = \int \frac{(u^2 - (u^2-1))du}{(u^2-1)u^2}$$

$$= \int \frac{du}{u^2-1} - \int \frac{du}{u^2}$$

$$= \frac{1}{2} \log \left| \frac{u-1}{u+1} \right| + \frac{1}{u} + C' \quad \left\{ \int \frac{du}{u^2-1} = \int \frac{du}{(u+1)(u-1)} \right.$$

$$I_1 = \frac{1}{2} \log \left| \frac{\cos x - 1}{\cos x + 1} \right| + \sec x + C' \quad \left. \begin{aligned} &= \frac{1}{2} \int \frac{(u+1) - (u-1)}{(u+1)(u-1)} du \\ &= \frac{1}{2} \log \left| \frac{u-1}{u+1} \right| \end{aligned} \right\}$$

$$\therefore I = \int \frac{(1-\sin x)dx}{\sin x(1+\sin x)} = \frac{1}{2} \log \left| \frac{\cos x - 1}{\cos x + 1} \right| + 2 \sec x - 2 \tan x + k$$

$\therefore$ where k is an arbitrary constant.

$$x = am^2$$

$$ay^2 = (am^2)^3$$

$$ay^2 = a^3 m^6$$

$$y^2 = a^2 m^6$$

$$\therefore y = \pm am^3$$

Considering $(x, y) = (am^2, am^3)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dm}}{\frac{dx}{dm}}$$

$$\frac{dy}{dm} = \frac{d(am^3)}{dm} = 3am^2$$

$$\frac{dx}{dm} = \frac{d(am^2)}{dm} = 2am$$

$$\therefore \left. \frac{dy}{dx} \right|_{(am^2, am^3)} = \frac{3am^2}{2am} = \frac{3m}{2} \quad \text{Slope of tangent at } (am^2, am^3)$$

$$\text{Slope of normal at } (am^2, am^3) = \frac{-1}{\left. \frac{dy}{dx} \right|_{(am^2, am^3)}} = \frac{-1}{\frac{3m}{2}} = \frac{-2}{3m}$$

Equation of normal to the curve at (am^2, am^3)

$$y - am^3 = \frac{-2}{3m} (x - am^2)$$

$$3my - 3am^4 = -2x + 2am^2$$

$$3my + 2x = 3am^4 + 2am^2 \text{ is the required equation.}$$

12

$$f(x) = \begin{cases} k \sin\left(\frac{\pi}{2}(x+1)\right) & x \leq 0 \\ \frac{\tan x - \sin x}{x^2} & x > 0 \end{cases}$$

$f(x)$ is continuous at $x=0$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$\text{Now } \lim_{x \rightarrow 0^-} f(x) = f(0) = \lim_{x \rightarrow 0^-} k \sin\left(\frac{\pi}{2}(x+1)\right) = k \sin \frac{\pi}{2} = k$$

$$\begin{aligned} \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} \frac{\tan x - \sin x}{x^2} = \lim_{x \rightarrow 0^+} \frac{\sin x}{x} \cdot \left(\frac{\sec x - 1}{x^2} \right) \\ &= \lim_{x \rightarrow 0^+} \frac{\sin x}{x} = \lim_{x \rightarrow 0^+} \frac{1 - \cos x}{x^2} \end{aligned}$$

$$= \lim_{x \rightarrow 0^+} \frac{\sin x}{x} \times \lim_{x \rightarrow 0^+} \frac{2 \sin^2 \frac{x}{2}}{4 \left(\frac{x}{2} \right)^2}$$

$$\lim_{x \rightarrow 0^+} \cos x$$

$$\text{Now } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \& \quad \lim_{x \rightarrow 0} \left(\frac{\sin(\frac{x}{2})}{(\frac{x}{2})} \right)^2 = 1$$

$$\therefore \lim_{x \rightarrow 0^+} f(x) = \frac{1 \times \frac{1}{2}}{1} = \frac{1}{2}$$

$$\therefore \boxed{k = \frac{1}{2}}$$

{ As $f(x)$ is continuous at $x=0$ }

13.

$$\text{Consider } \tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$$

$$= \tan^{-1} \left(\frac{1 - \sqrt{\frac{1-x^2}{1+x^2}}}{1 + \sqrt{\frac{1-x^2}{1+x^2}}} \right)$$

$$\text{Put } \theta = \cos^{-1} x$$

$$\therefore \theta = \frac{1}{2} \cos^{-1} x^2$$

$$\therefore \tan^{-1} \left(\frac{1 - \sqrt{\frac{1-x^2}{1+x^2}}}{1 + \sqrt{\frac{1-x^2}{1+x^2}}} \right) = \tan^{-1} \left(\frac{1 - \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}}{1 + \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}} \right)$$

$$1 - \cos 2\theta = 2\sin^2 \theta$$

$$1 + \cos 2\theta = 2\cos^2 \theta$$

$$\therefore \tan^{-1} \left(\frac{1 - \sqrt{\frac{1-x^2}{1+x^2}}}{1 + \sqrt{\frac{1-x^2}{1+x^2}}} \right) = \tan^{-1} \left(\frac{1 - \tan \theta}{1 + \tan \theta} \right) = \tan^{-1} \left(\frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \tan \theta} \right)$$

$$= \tan^{-1} \left(\tan \left(\frac{\pi}{4} - \theta \right) \right)$$

We want

$$\frac{d}{d(\cos^2 x)} \left(\tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right) \right) = \frac{d}{d\left(\frac{\pi}{4} - \theta\right)} \left(\frac{\pi}{4} - \theta \right) = \frac{\frac{d}{d\theta} \left(\frac{\pi}{4} - \theta \right)}{\frac{d}{d\theta} (2\theta)}$$

$$= \frac{1}{2} \frac{d}{d\theta} \left(\frac{\pi}{4} - \theta \right) = -\frac{1}{2}$$

14. A plane which passes through $A(3, 2, 1)$, $B(4, 2, -2)$ & $C(6, 5, -1)$

is
$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 4-3 & 2-2 & -2-1 \\ 6-3 & 5-2 & -1-1 \end{vmatrix} = 0$$

$$0 = \begin{vmatrix} x-3 & y-2 & z-1 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix}$$

$$0 = (x-3)(0 \times (-2) - (-3) \times 3) - (y-2)(1 \times (-2) - (-3) \times 3) + (z-1)(1 \times 3 - 3 \times 0)$$

$$9(x-3) - 7(y-2) + 3(z-1) = 0$$

$$9x - 7y + 3z = 27 - 14 + 3 = 16$$

$\therefore$ Plane passing through points A, B, C is $9x - 7y + 3z = 16$

Now A, B, C & $D(\lambda, 5, 5)$ are coplanar

$\therefore D$ lies on the plane of A, B, C

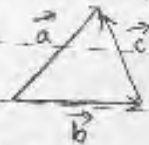
$$9\lambda - 7 \times 5 + 3 \times 5 = 16$$

$$9\lambda = 36$$

$$\lambda = 4$$

15.

$$\vec{a} = \vec{b} + \vec{c}$$



Here

$$\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}$$

$$\vec{b} = s\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{a} = \vec{b} + \vec{c}$$

$$p\hat{i} + q\hat{j} + r\hat{k} = s\hat{i} + 3\hat{j} + 4\hat{k} + 3\hat{i} + \hat{j} - 2\hat{k}$$

$$p\hat{i} + q\hat{j} + r\hat{k} = (s+3)\hat{i} + 4\hat{j} + 2\hat{k}$$

Equating components,

$$p = s+3$$

$$q = 4$$

$$r = 2$$

Area of triangle = $5\sqrt{6}$

$$\text{But Area of triangle} = \frac{1}{2} |\vec{a} \times \vec{b}|$$

$$5\sqrt{6} = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ p & 4 & 2 \\ p-3 & 3 & 4 \end{vmatrix} = \frac{1}{2} \left| 10\hat{i} - (2p+6)\hat{j} + (12-p)\hat{k} \right|$$

$$600 = 100 + (2p+6)^2 + (12-p)^2$$

$$500 = 4p^2 + 36 + 24p + 144 + p^2 - 24p \Rightarrow 5p^2 = 320$$

$$\frac{360}{40} = 9$$

$$p^2 = 64$$

$$p = \pm 8$$

$$\text{If } p = 8, \quad S = p - 3 = 5$$

$$\text{If } p = -8, \quad S = p - 3 = -11$$

16.



let E_1 : Event that the balls are drawn from bag A

E_2 : Event that the balls are drawn from Bag B

C: Event that 2 white balls & 1 red ball are drawn

$$P(E_2|C) = ?$$

$$\text{Now } P(E_1) = \frac{1}{2} = P(E_2)$$

$$P(C|E_1) = \frac{3}{7} \times \frac{2}{6} \times \frac{4}{5} \times 3 = \frac{4}{35} \times 3$$

$$P(C|E_2) = \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} \times 3 = \frac{6}{35} \times 3$$

(multiplied by 3 because they can be chosen in any order)

By Bayes theorem,

$$P(E_2|C) = \frac{P(E_2)P(C|E_2)}{P(E_1)P(C|E_1) + P(E_2)P(C|E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{6}{35} \times 3}{\frac{1}{2} \times \frac{4}{35} \times 3 + \frac{1}{2} \times \frac{6}{35} \times 3} = \frac{6}{10} = \frac{3}{5}$$

17.

Let the length of the plot be l .

Let the breadth of the plot be b .

Now $l \times b = A$ ($A = \text{Area of the plot}$)

$$(l-50)(b+50) = A$$

$$(l-10)(b-20) = A - 5300$$

$$lb - 50b + 50l - 2500 = A$$

$$-b + l = 50 \quad \text{--- (1) } (A = 20)$$

$$lb - 10b - 20l + 200 = A - 5300$$

$$10b + 20l = 5500$$

$$b + 2l = 550 \quad \text{--- (2)}$$

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} l \\ b \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$\begin{matrix} \text{"} & \text{"} & \text{"} \\ A & X & B \end{matrix}$

$$AX = B$$

$$\therefore X = A^{-1}B$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$(ad-bc = 1 \times 1 - (-2) \times 1 = 3 \neq 0)$$

$\therefore A^{-1}$ exists

Here $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$

$$A^{-1} = \frac{1}{1+2} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 550 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 600 \\ 450 \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$$

$$\begin{bmatrix} l \\ b \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$$

$$l = 200 \text{ m}$$

$$b = 150 \text{ m}$$

He wants to donate the plot to the school because he wants rural places to become developed & he is thereby showing his kind heartedness & his intention to help the society to develop by producing more literates. Children should have an opportunity to learn.

$$2ye^{\frac{x}{y}} dx + (y - 2xe^{\frac{x}{y}}) dy = 0$$

$$\frac{dy}{dx} = \frac{-2ye^{\frac{x}{y}}}{y - 2xe^{\frac{x}{y}}}$$

$$\frac{dx}{dy} = \frac{y - 2xe^{\frac{x}{y}}}{-2ye^{\frac{x}{y}}} = f(x, y) \text{ (say)}$$

$$F(\lambda x, \lambda y) = \frac{\lambda y - 2\lambda x e^{\frac{\lambda x}{\lambda y}}}{-2\lambda y e^{\frac{\lambda x}{\lambda y}}} = \lambda^0 F(x, y)$$

$\therefore$ The equation is a homogeneous differential equation

Put $x = vy$

$$\frac{dx}{dy} = v + y \frac{dv}{dy}$$

$$\therefore v + y \frac{dv}{dy} = \frac{y - 2vy e^v}{-2y e^v} = -\frac{1}{2} e^{-v} + v \Rightarrow y \frac{dv}{dy} = -\frac{1}{2} e^{-v}$$

$$\therefore e^y dy = \frac{-1}{2} \frac{dy}{y}$$

Integrating

$$e^y = -\frac{1}{2} \log y + C$$

$\therefore e^{\frac{y}{2}} = -\frac{1}{2} \log y + C$ is the required solution to the differential equation

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$$(x+1) \frac{dy}{dx} - y = e^{3x} (x+1)^3$$

$$\frac{dy}{dx} - \frac{y}{(x+1)} = e^{3x} (x+1)^2$$

This equation is of the form $\frac{dy}{dx} + Py = Q$ { P, Q = functions of x alone }
linear differential equation.

$$\therefore \text{Integrating factor} = e^{\int P dx} = e^{-\int \frac{dx}{x+1}} = e^{-\log(x+1)} = \frac{1}{x+1}$$

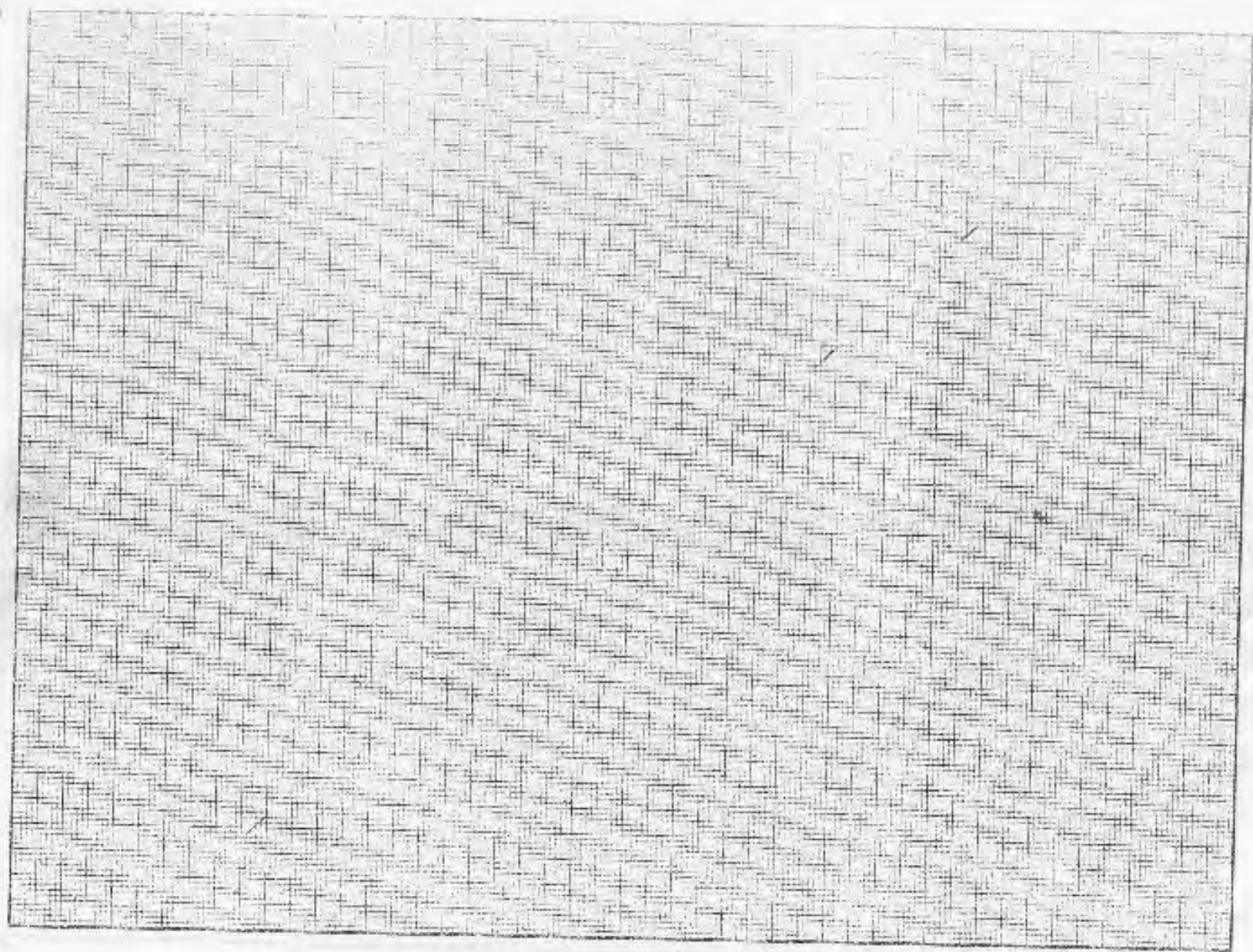
Solution to the differential equation:

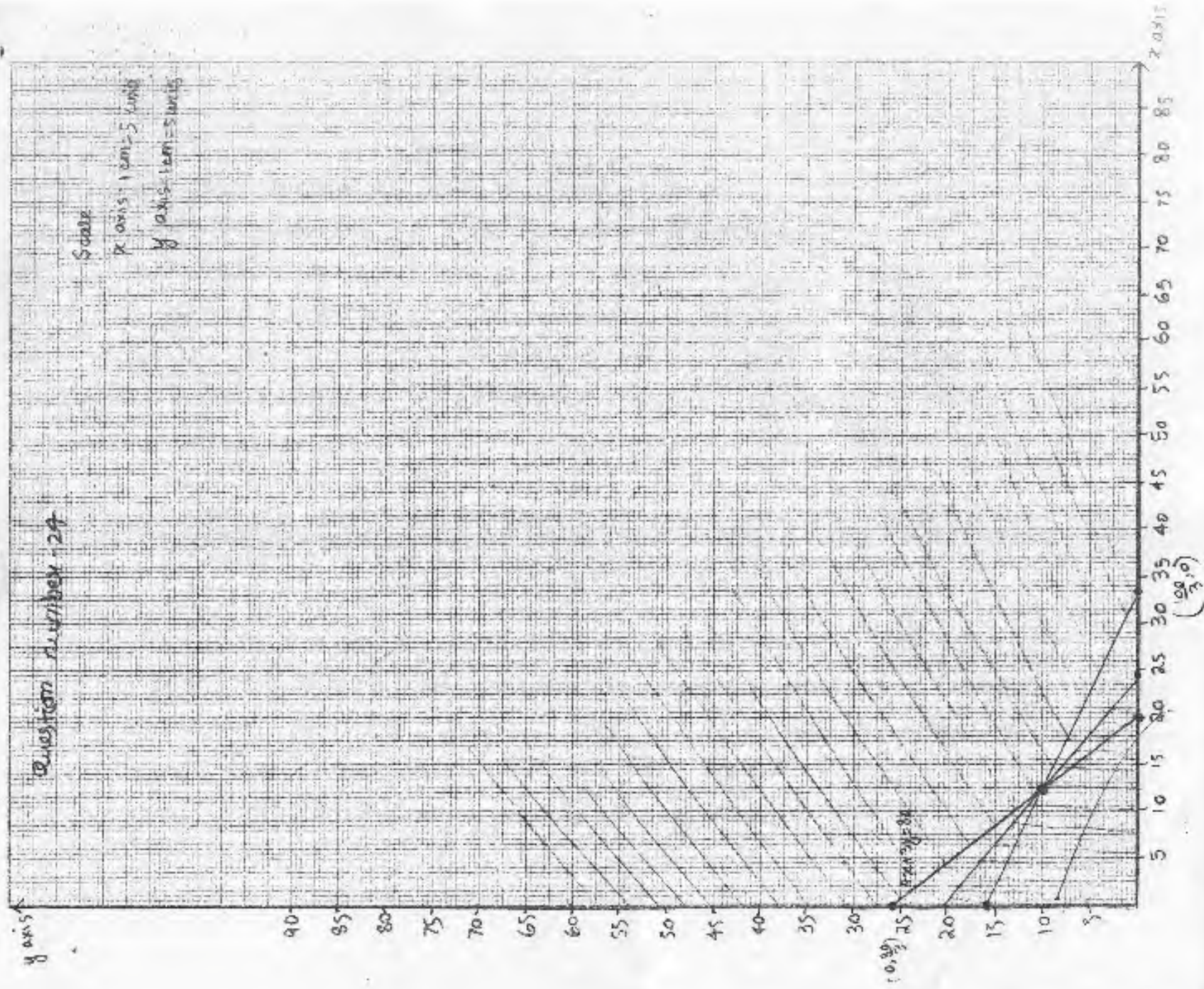
$$y \times \text{I.F.} = \int Q \times \text{I.F.} dx + C$$

$$\therefore \frac{y}{x+1} = \int e^{3x} (x+1)^2 \cdot \frac{1}{(x+1)} dx + C$$

$$= \int x e^{3x} dx + \int e^{3x} dx + C = \frac{x e^{3x}}{3} - \frac{1}{9} \int e^{3x} dx + \int e^{3x} dx + C$$

$$\frac{y}{x+1} = \frac{x e^{3x}}{3} + \frac{2}{9} e^{3x} + C \text{ is the solution to the required differential equation}$$





Section-C

24. let the number of units of ^{food F₁} Vitamin A = x
 let the number of units of ^{food F₂} Vitamin Minerals = y

$$x \geq 80 \quad x \geq 0 \quad x \geq 0 \quad y \geq 0$$

$$y \geq 100 \quad y \geq 0 \quad 4x + 3y \geq 80$$

$$3x + 6y \geq 100$$

$$4x + 3y = 80$$

$$3x + 6y = 100$$

$$8x + 6y = 160$$

$$5x = 60$$

$$x = 12$$

$$y = \frac{32}{3}$$

$$Z = 5x + 6y$$

Minimize Z

(x, y)

Z

$$(0, \frac{80}{3})$$

$$₹. 160$$

$$(\frac{100}{3}, 0)$$

$$₹. 166.67$$

$$(12, \frac{32}{3})$$

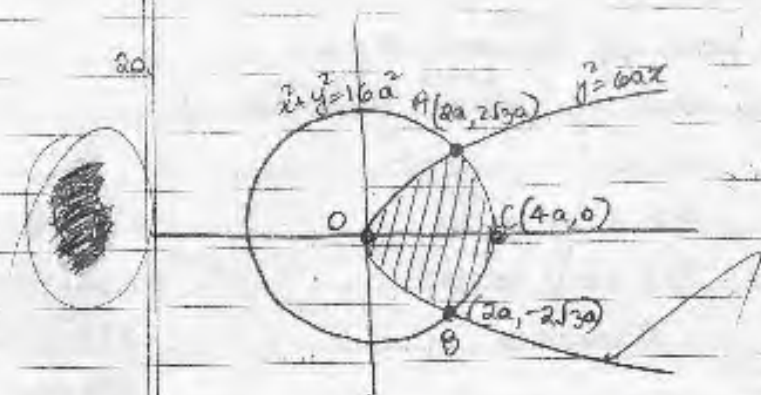
$$₹. 124$$

Since the region is unbounded, we should check if any point other than $(12, \frac{32}{3})$ is in common to region shaded, & $5x + 6y \leq 124$

Clearly, from the graph, only $(12, \frac{32}{3})$ lies on both regions.

∴ The Minimum cost of the diet = ₹. 124

with 12 units of food F₁ & $\frac{32}{3}$ units of food F₂



Solving the two curves

$$x^2 + 6ax = 16a^2$$

$$\left(\frac{x}{a}\right)^2 + 6\left(\frac{x}{a}\right) - 16 = 0$$

$$\left(\frac{x}{a} + 8\right)\left(\frac{x}{a} - 2\right) = 0$$

$$x = -8a \text{ or } x = 2a$$

x

$$\therefore x = 2a$$

$$y = \pm 2\sqrt{3}a \quad (y^2 = 12a^2 \quad y = \pm 2\sqrt{3}a)$$

$$\therefore \text{Required Area} = 2 \times \text{Area OACO}$$

$$= 2 \times \int_{2\sqrt{3}a}^{2\sqrt{3}a} (x_1 - x_2) dy$$

$$= 2 \times \int_0^{2\sqrt{3}a} \sqrt{16a^2 - y^2} dy = 2 \times \int_0^{2\sqrt{3}a} \frac{y^2}{6a} dy$$

$$= 2 \times \left[\frac{y}{2} \sqrt{16a^2 - y^2} + \frac{16a^2}{2} \sin^{-1} \frac{y}{4a} \right]_0^{2\sqrt{3}a} = \left[\frac{y^3}{9a} \right]_0^{2\sqrt{3}a}$$

$$= 2\sqrt{3}a \times 2a + 16a^2 \sin^{-1} \frac{\sqrt{3}}{2} - 0 = \frac{8 \times 2\sqrt{3}a^2}{\sqrt{3}}$$

$$= 4\sqrt{3}a^2 = \frac{8\sqrt{3}a^2}{3} + \frac{16a^2\pi}{3} \quad \left\{ \int \sqrt{A^2 - x^2} dx = \frac{x}{2} \sqrt{A^2 - x^2} + \frac{A^2}{2} \sin^{-1} \frac{x}{A} \right\}$$

$$= \frac{4}{\sqrt{3}}a^2 + \frac{16a^2\pi}{3} \quad \text{sq units}$$

20. $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$

$f(x)$ is increasing strictly in an interval if $f'(x) > 0$ in that interval

$f(x)$ is strictly decreasing in an interval if $f'(x) < 0$ in that interval

$$f'(x) = 4x^3 - 24x^2 + 44x - 24$$

$$= 4(x^3 - 6x^2 + 11x - 8)$$

21.

Maximum & Minimum values of $f(x) = \sec x + \log \cos x$
 ~~$\sec x + \log \cos x$~~

$$f'(x) = 0$$

$$\sec x \tan x + \frac{2}{\cos x} (-\sin x) = 0 \quad x \neq \frac{\pi}{2}$$

$$\therefore \tan x (\sec x - 2) = 0$$

$$\therefore x = \pi \quad \text{or} \quad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

Now $f'(x) < 0$ for maximum

$f'(x) > 0$ for minimum

$$f''(x) = \sec^2 x (\sec x - 2) + \tan^2 x \sec x$$

$$f''(\pi) = 1 \times -3 + 0 = -3 < 0$$

$$f''\left(\frac{\pi}{3}\right) = f''\left(\frac{5\pi}{3}\right) = 0 + 3 \times 2 = 6 > 0$$

Function attains maximum value at $x = \pi$ & minimum

value at $x = \frac{\pi}{3}, \frac{5\pi}{3}$

$$f(\pi) = -1$$

$$f\left(\frac{\pi}{3}\right) = 2 - 2 \log 2$$

But when $x = \frac{\pi}{2}$, function becomes undefined.

Minimum & Maximum do not exist at Minimum.

22 TP: $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ac \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

Proof :

$$C_1 \rightarrow C_1 - 2C_3$$

$$\Delta = \begin{vmatrix} b^2+c^2 & a^2 & bc \\ c^2+a^2 & b^2 & ac \\ a^2+b^2 & c^2 & ab \end{vmatrix}$$

$$C_1 \rightarrow C_1 + C_2$$

Now taking $a^2+b^2+c^2$ common from C_1 ,

$$\Delta = (a^2+b^2+c^2) \begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ac \\ 1 & c^2 & ab \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\Delta = (a^2 + b^2 + c^2) \begin{vmatrix} 1 & a^2 & bc \\ 0 & b^2 - a^2 & ac - bc \\ 0 & c^2 - a^2 & ab - bc \end{vmatrix}$$

Taking $(a-b)$ common from R_2 & $(c-a)$ common from C_3

$$\Delta = (a^2 + b^2 + c^2) (a-b)(c-a) \begin{vmatrix} 1 & a^2 & bc \\ 0 & -(a+b) & c \\ 0 & a+c & -b \end{vmatrix}$$

Expanding along C_1

$$\Delta = (a^2 + b^2 + c^2) (a-b)(c-a) (- (a+b) \times b + (a+c)c)$$

$$= (a^2 + b^2 + c^2) (a-b)(c-a) (b^2 - c^2 + bc - ba)$$

$$= (a^2 + b^2 + c^2) (a-b)(b-c)(c-a)(a+b+c)$$

$$\text{As } (b-c)(a+b+c) = ab + b^2 + bc - ac - bc - c^2$$

$$b^2 - c^2 + ab - ac$$

25.

Equation of plane containing two parallel lines
has DRS of perpendicular

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b}$$

$\vec{b}$ = DRS of the line

$\vec{a}_1$ = Point (Position vector on line 1)

$\vec{a}_2$ = (Position vector) on line 2

$$\vec{a}_1 = \hat{i} - \hat{j}$$

$$\vec{a}_2 = 2\hat{j} - \hat{k}$$

$$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = -\hat{i} + 3\hat{j} - \hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix} = 8\hat{i} + \hat{j} - 5\hat{k}$$

$$\therefore \text{Equation of plane} = 8x + y - 5z = 8 \times 1 + \hat{j} \times (-1) - 5 \times 0$$

$$8x + y - 5z = 7$$

$$\text{Now consider the line } \frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{2}$$

$$8 \times 3 + 1 \times 1 + -5 \times 5 = 0$$

$\therefore$ DRS of line & perpendicular to plane are perpendicular.
& Point on line = (2, 1, 2) $8 \times 2 + 1 - 5 \times 2 = 7$ also satisfies the
equation of plane. $\therefore$ The line is contained in the plane.

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$$f(x) = 4x^2 + 12x + 15$$

$$\text{Let } f(x) = y$$

$$y = 4x^2 + 12x + 15$$

$$4x^2 + 12x + (15 - y) = 0$$

$$x = \frac{-12 \pm \sqrt{144 - 16(15 - y)}}{8} = \frac{-12 \pm \sqrt{9 - 5 + y}}{8}$$

$$x = \frac{\sqrt{y - 6} - 3}{2}$$

$$\text{Consider } g(x) = \frac{\sqrt{x - 6} - 3}{2}$$

$$\begin{aligned} \text{Now } fog(x) &= f(g(x)) = f\left(\frac{\sqrt{x - 6} - 3}{2}\right) = \frac{4}{4}(\sqrt{x - 6} - 3)^2 + 6(\sqrt{x - 6} - 3) + 15 \\ &= x - 6 + 9 - 6\sqrt{x - 6} + 6\sqrt{x - 6} - 18 + 15 \\ &= x \end{aligned}$$

$$gof(x) = g(f(x)) = g(4x^2 + 12x + 15) = \frac{\sqrt{4x^2 + 12x + 9} - 3}{2} = \frac{2x + 3 - 3}{2} = x$$

$$\therefore fog(x) = gof(x) = x$$

By definition, g is the inverse of f .

As inverse exists, f is invertible &

$$\Rightarrow f^{-1}(x) = \frac{\sqrt{x-6}-3}{2}$$

$$f^{-1}(31) = \frac{\sqrt{31-6}-3}{2} = \frac{5-3}{2} = 1$$

$$f^{-1}(87) = \frac{\sqrt{87-6}-3}{2} = \frac{9-3}{2} = 3$$

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$X =$	1	2	3	4	5	6
$P(X=)$	$\frac{10}{20} = \frac{1}{2}$	$\frac{6}{20} = \frac{3}{10}$	$\frac{3}{20}$	$\frac{1}{20}$	0	0

(as 5 & 6 can't be minimum values)

$$P(X=1) = \frac{{}^5C_2}{{}^6C_3} = \frac{10}{20}$$

(Numbers from 2-6 can be chosen)
(Any two)

$$P(X=2) = \frac{{}^4C_2}{{}^6C_3} = \frac{6}{20}$$

(Numbers from 2-6 can be chosen)
(Any two)

$$P(X=3) = \frac{{}^3C_2}{{}^6C_3} = \frac{3}{20}$$

(Any 2 out of 4, 3, 6 can be chosen)

$$P(X=4) = \frac{{}^2C_2}{{}^6C_3} = \frac{1}{20}$$

only 5, 6 can be chosen

$$\text{Mean} = E(x) = \sum x_i p_i$$

$$= \frac{10}{20} \times 1 + \frac{6}{20} \times 2 + \frac{3}{20} \times 3 + \frac{1}{20} \times 4 + 0 + 0$$

$$= \frac{10}{20} + \frac{12}{20} + \frac{9}{20} + \frac{4}{20} = \frac{35}{20} = 1.75$$

$$\text{Variance} = E(x^2) - (E(x))^2$$

$$= \frac{10}{20} \times 1 + \frac{6}{20} \times 4 + \frac{3}{20} \times 9 + \frac{1}{20} \times 16 - \frac{49}{16}$$

$$= \frac{10 + 24 + 27 + 16}{20} - \frac{49}{16} = \frac{77}{20} - \frac{49}{16}$$

$$= \frac{7}{4} \left(\frac{-11}{5} - \frac{7}{4} \right)$$

$$= \frac{7}{4} \times \frac{9}{20}$$

$$\sigma^2 = \frac{63}{80}$$

$$\frac{44}{9}$$