DAY — 09

SEAT NUMBER

2022	III	14	1030	J-762	(E)

MATHEMATICS & STATISTICS (40)
(ARTS & SCIENCE)

Time : 3 Hrs. (7 Pages) Max. Marks : 80

General instructions :

The question paper is divided into ***FOUR*** *sections.*

(1) **Section A :** *Q. 1 contains* ***Eight*** *multiple choice type of questions, each carrying* ***Two*** *marks.*
Q. 2 contains ***Four*** *very short answer type questions, each carrying* ***one*** *mark.*

(2) **Section B :** *Q. 3 to Q. 14 contain* ***Twelve*** *short answer type questions, each carrying* ***Two*** *marks. (Attempt any* ***Eight****)*

(3) **Section C :** *Q. 15 to Q. 26 contain* ***Twelve*** *short answer type questions, each carrying* ***Three*** *marks. (Attempt any* ***Eight****)*

(4) **Section D :** *Q. 27 to Q. 34 contain* ***Eight*** *long answer type questions, each carrying* ***Four*** *marks. (Attempt any* ***Five****)*

(5) *Use of log table is allowed. Use of calculator is not allowed.*

(6) *Figures to the right indicate full marks.*

(7) *Use of graph paper is not necessary. Only rough sketch of graph is expected.*

(8) *For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet, e.g., (a) ..../ (b) ..../ (c) ..../ (d)........., etc. No marks shall be given, if ONLY the correct answer or the alphabet of correct answer is written. Only the first attempt will be considered for evaluation.*

(9) *Start answer to each section on a new page.*

SECTION – A

Q. 1. **Select and write the correct answer for the following multiple choice type of questions :** [16]

(i) The negation of $p \wedge (q \to r)$ is ______.

(a) $\sim p \wedge (\sim q \to \sim r)$ (b) $p \vee (\sim q \vee r)$

(c) $\sim p \wedge (\sim q \to r)$ (d) $p \to (q \wedge \sim r)$ (2)

(ii) In ΔABC if $c^2 + a^2 - b^2 = ac$, then $\angle B =$ ______.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$ (2)

(iii) Equation of line passing through the points (0, 0, 0) and (2, 1, –3) is ______.

(a) $\frac{x}{2} = \frac{y}{1} = \frac{z}{-3}$ (b) $\frac{x}{2} = \frac{y}{-1} = \frac{z}{-3}$

(c) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ (d) $\frac{x}{3} = \frac{y}{1} = \frac{z}{2}$ (2)

(iv) The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is ______.

(a) 0 (b) – 1

(c) 1 (d) 3 (2)

(v) If $f(x) = x^5 + 2x - 3$, then $(f^{-1})^1 (-3) =$ ______.

(a) 0 (b) – 3

(c) $-\frac{1}{3}$ (d) $\frac{1}{2}$ (2)

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(vi) The maximum value of the function $f(x) = \frac{\log x}{x}$ is ______.

(a) e (b) $\frac{1}{e}$

(c) e^2 (d) $\frac{1}{e^2}$ (2)

(vii) If $\int \frac{dx}{4x^2 - 1} = A \log\left(\frac{2x-1}{2x+1}\right) + c$, then A = ______.

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$ (2)

(viii) If the p.m.f. of a r.v. X is

$P(x) = \frac{c}{x^3}$, for $x = 1, 2, 3$

$= 0$, otherwise,

then $E(X) =$ ______.

(a) $\frac{216}{251}$ (b) $\frac{294}{251}$

(c) $\frac{297}{294}$ (d) $\frac{294}{297}$ (2)

Q. 2. Answer the following questions : **[4]**

(i) Find the principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$. (1)

(ii) Write the separate equations of lines represented by the equation $5x^2 - 9y^2 = 0$ (1)

(iii) If $f'(x) = x^{-1}$, then find $f(x)$ (1)

(iv) Write the degree of the differential equation $(y''')^2 + 3(y'') + 3xy' + 5y = 0$ (1)

SECTION – B

Attempt any EIGHT of the following questions : **[16]**

Q. 3. Using truth table verify that :

$(p \wedge q) \vee \sim q \equiv p \vee \sim q$ **(2)**

Q. 4. Find the cofactors of the elements of the matrix $\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$ **(2)**

Q. 5. Find the principal solutions of $\cot \theta = 0$ **(2)**

Q. 6. Find the value of k, if $2x + y = 0$ is one of the lines represented by $3x^2 + kxy + 2y^2 = 0$ **(2)**

Q. 7. Find the cartesian equation of the plane passing through A(1, 2, 3) and the direction ratios of whose normal are 3, 2, 5. **(2)**

Q. 8. Find the cartesian co-ordinates of the point whose polar co-ordinates are $\left(\frac{1}{2}, \frac{\pi}{3}\right)$. **(2)**

Q. 9. Find the equation of tangent to the curve $y = 2x^3 - x^2 + 2$ at $\left(\frac{1}{2}, 2\right)$. **(2)**

Q. 10. Evaluate : $\int_0^{\frac{\pi}{4}} \sec^4 x \, dx$ **(2)**

Q. 11. Solve the differential equation $y\frac{dy}{dx} + x = 0$ **(2)**

Q. 12. Show that function $f(x) = \tan x$ is increasing in $\left(0, \frac{\pi}{2}\right)$. **(2)**

Q. 13. Form the differential equation of all lines which makes intercept 3 on x-axis. (2)

Q. 14. If $X \sim B(n, p)$ and $E(X) = 6$ and Var $(X) = 4.2$, then find n and p. (2)

SECTION – C

Attempt any EIGHT of the following questions : [24]

Q. 15. If $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$, then find the value of x. (3)

Q. 16. If angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between the lines represented by $2x^2 - 5xy + 3y^2 = 0$, then show that $100(h^2 - ab) = (a + b)^2$. (3)

Q. 17. Find the distance between the parallel lines $\frac{x}{2} = \frac{y}{-1} = \frac{z}{2}$ and $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z-1}{2}$ (3)

Q. 18. If A $(5, 1, p)$, B$(1, q, p)$ and C$(1, -2, 3)$ are vertices of a triangle and $G\left(r, \frac{-4}{3}, \frac{1}{3}\right)$ is its centroid, then find the values of p, q, r by vector method. (3)

Q. 19. If A$(\bar{a})$ and B$(\bar{b})$ be any two points in the space and R$(\bar{r})$ be a point on the line segment AB dividing it internally in the ratio $m : n$ then prove that $\bar{r} = \frac{m\bar{b} + n\bar{a}}{m+n}$. (3)

Q. 20. Find the vector equation of the plane passing through the point A$(-1, 2, -5)$ and parallel to the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$. (3)

Q. 21. If $y = e^{m\tan^{-1}x}$, then show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-m)\frac{dy}{dx} = 0$ (3)

Q. 22. Evaluate : $\int \frac{dx}{2+\cos x - \sin x}$ (3)

Q. 23. Solve $x + y\frac{dy}{dx} = \sec(x^2+y^2)$ (3)

Q. 24. A wire of length 36 meters is bent to form a rectangle. Find its dimensions if the area of the rectangle is maximum. (3)

Q. 25. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X. (3)

Q. 26. If a fair coin is tossed 10 times. Find the probability of getting at most six heads. (3)

SECTION – D

Attempt any FIVE of the following questions : **[20]**

Q. 27. Without using truth table prove that

$(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$ (4)

Q. 28. Solve the following system of equations by the method of inversion

$x - y + z = 4,\ 2x + y - 3z = 0,\ x + y + z = 2$ (4)

Q. 29. Using vectors prove that the altitudes of a triangle are concurrent. (4)

Q. 30. Solve the L. P. P. by graphical method,

Minimize $z = 8x + 10y$

Subject to $2x + y \geq 7,$

$2x + 3y \geq 15,$

$y \geq 2,\ x \geq 0$ (4)

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Q. 31. If $x = f(t)$ and $y = g(t)$ are differentiable functions of t so that y is differentiable function of x and $\frac{dx}{dt} \neq 0$, then prove that :

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Hence find $\frac{dy}{dx}$ if $x = \sin t$ and $y = \cos t$. (4)

Q. 32. If u and v are differentiable functions of x, then prove that :

$$\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$$

Hence evaluate $\int \log x\,dx$ (4)

Q. 33. Find the area of region between parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (4)

Q. 34. Show that : $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx = \frac{\pi}{8}\log 2$ (4)

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