

BT-4/J-21

44151

DISCRETE MATHEMATICS

Paper-PC-CS-202A

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *five* questions in all, selecting at least *one* question from each unit.

UNIT-I

1. (a) Show that

$$1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

by mathematical induction.

- (b) Given that

$$(A \cap C) \subseteq (B \cap C)$$

$$(A \cap \bar{C}) \subseteq (B \cap \bar{C})$$

show that $A \subseteq B$.

2. (a) Construct the truth tables for the following statements

(i) $(p \rightarrow p) \rightarrow (p \rightarrow \bar{p})$.

(ii) $(p \vee \bar{q}) \rightarrow \bar{p}$.

(iii) $p \leftrightarrow (\bar{p} \vee \bar{q})$.

- (b) Let A, B and C be sets such that $(A \cap B \cap C) = \phi$, $(A \cap B) \neq \phi$, $(A \cap C) \neq \phi$ and $(B \cap C) \neq \phi$. Draw the corresponding Venn diagram.

UNIT-II

3. (a) Find all the partitions of $B = \{a, b, c, d\}$.
- (b) Let $A = \{a, b\}$ and $B = \{4, 5, 6\}$. Given each of the following :
- (i) $A \times B$
 - (ii) $B \times A$
 - (iii) $A \times A$
 - (iv) $B \times B$.
4. (a) Show that if R_1 and R_2 are equivalence relations on A , then $R_1 \cap R_2$ is an equivalence relation on A .
- (b) Let $A = \mathbb{Z}$, the set of integers and let R is defined by $a R b$ if and only if $a \leq b$. Is R is an equivalence relation ?

UNIT-III

5. (a) Prove that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are one-to-one functions, then $g \circ f$ is one-to-one.
- (b) Let $A = B = C = \mathbb{R}$, and let $f: A \rightarrow B$ and $g: B \rightarrow C$ be defined by $f(a) = a - 1$ and $g(b) = b^2$ find :
- (i) $(f \circ g)(2)$
 - (ii) $(g \circ f)(2)$
 - (iii) $(f \circ f)(y)$
 - (iv) $(g \circ g)(y)$.

6. (a) Let A and B be two finite set with same number of elements, and let $f: A \rightarrow B$ be an everywhere defined functions :
- (i) If f is one-to-one, then f is onto.
 - (ii) If f is onto, then f is one-to-one.
- (b) If n pigeons are assigned to m pigeonholes, and $m < n$, then atleast one pigeonhole contains two or more pigeons.

UNIT-IV

7. (a) Define the following :
- (i) Group.
 - (ii) Cyclic group.
- (b) Let H and K be subgroups of group G :
- (i) Prove that $H \cap K$ is subgroup of G .
 - (ii) Show that $H \cup K$ need not be subgroup of G .
8. (a) Let G be an Abelian group and n is a fixed integer. Prove that the function $f: G \rightarrow G$ defined by $f(a) = a^n$, for $a \in G$ is a homomorphism.
- (b) Let G be a group, and let $H = \{x/x \in G \text{ and } ax = xa \text{ for all } a \in G\}$. Show that H is a normal subgroup of G .
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