Roll No. .... Total Pages : 04

BT-3/D-19 33080

DISCRETE STRUCTURE

CSE-201N/IT-209N

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit.

Unit I

1. (a) Let P, Q and R are three finite sets. Then prove that :

$$|P \cup Q \cup R| = |P| + |Q| + |R| - |P \cap Q| - |P \cap R| - |Q \cap R| + |P \cap Q \cap R|$$

Also draw Venn diagram. **8**

(b) Let A and B be two sets, then show that $(A \cup B)^c = A^c \cap B^c$. Also justify your answer by giving suitable example. **7**

2. (a) Determine which propositions are the following by constructing Truth Tables :

(i) $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$

(ii) $(p \leftrightarrow q) \to ((p \wedge q) \vee (\sim p \wedge q))$ **8**

(b) Prove by Mathematical Induction for any integer n, $11^{n+2} + 12^{2n+1}$ is divisible by 133. 7

Unit II

3. (a) Let A = $\{a, b, c, d\}$. Let R = $\{(a, b), (a, c), (b, a), (b, c) (c, d), (d, a)\}$. Find the Reflexive-transitive closure of R. 8

(b) Find whether the relation $(x, y) \in R$, if $x \geq y$ defined on the set of positive integers is a partial order relation or not. 7

4. Let $D_{100} = \{1, 2, 4, 5, 10, 20, 25, 50, 100\}$ and let the relation $R(\leq)$ be the relation (divides) a partial ordering on D_{100}.

(a) Draw the Hasse Diagram for the above relation :

(i) Determine the GLB of B, where B = {10, 20}

(ii) Determine the LUB of B, where B = {10, 20}

(iii) Determine the GLB of B, where B = {5, 10, 20, 25}

(iv) Determine the LUB of B, where B = {5, 10, 20, 25}. 10

(b) Determine whether (D_{100}, R) is a lattice or not. 5

Unit III

5. (a) In a shipment, there are 40 floppy disks of which 5 are defective. Determine :

   (i) In how many ways can we select 5 floppy disks ?

   (ii) In how many ways can we select 5 non-defective floppy disks ?

   (iii) In how many ways can we select 5 floppy disks containing exactly 3 defective floppy disks ? **7**

   (b) How many permutations can be made out of the letters of word "COMPUTER" ? How many of these :

   (i) Begin with C

   (ii) End with R

   (iii) Begin with C and end with R

   (iv) C and R occupy the end places. **8**

6. (a) Solve the recurrence relation $a_{r+2} - 2a_{r+1} + a_r = 2^r$ by the method of generating functions with the initial conditions $a_0 = 2$ and $a_1 = 1$. **8**

   (b) Find the particular solution of the difference equation $a_{r+2} - 4a_r = r^2 + r + 1$. **7**

Unit IV

7\. (a) Consider an algebraic system (G, *), where 'G' is the set of all non-zero real numbers and '*' is a binary operation defined by $a*b - ab/4$. Show that (G, *) is an abelian group. **8**

(b) Let (I, +) be a group, where I is the set of all integers and '+' is an addition operation. Determine whether the following subsets of G are subgroups of G :

(i) The set G_1 of all odd integers

(ii) The set G_2 of all positive integers. **7**

8\. (a) Consider an algebraic system (Q, *), where 'Q' is the set of all rational numbers and '*' is a binary operation defined by $a * b = a + b - ab$ for all $a, b \in Q$. Determine whether (Q, +) is a group or not. **8**

(b) Explain Ring Homomorphism with example. **7**