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BT-4 / M-18

MATHEMATICS - III

Paper–MATH–201E

Time allowed : 3 hours]   *[Maximum marks : 100*

Note :- *Attempt any **five** questions selecting at least **one** from each section.*

Section–I

1. (a) Develop $\sin\left(\dfrac{\pi x}{\ell}\right)$ in half-range cosine series in the range $0 < x < \ell$.   10

   (b) Find the fourier transform of the following function.   10

   $$f(x)=\begin{cases} x^2, & 0 \le x \le \pi \\ -x^2, & -\pi \le x \le 0 \end{cases}$$

2. (a) Find the fourier cosine transform of   10
   $$f(x)=\frac{1}{1+x^2}$$

   (b) Solve $\dfrac{\partial V}{\partial t} = K\dfrac{\partial^2 V}{\partial x^2}$ for $x > 0, t > 0$ under the boundary conditions $V = V_0$ when $x = 0, t > 0$ and the initial condition $V = 0$, when $t = 0, x > 0$ 10

Section –II

3. (a) Prove that   10

   $$\log(1 + re^{i\theta}) = \frac{1}{2}\log(1 + 2r\cos\theta + r^2) + i\tan^{-1}\left(\frac{r\sin\theta}{1+r\cos\theta}\right)$$

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(b) Show that   10

   (i) $\sinh^{-1}(\tan\theta) = \log\tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right)$

   (ii) $\operatorname{sech}^{-1}(\sin\theta) = \log\cot\dfrac{\theta}{2}$

4. (a) In a two-dimensional fluid flow, the stream function ψ is given, find the velocity potential ϕ.

   $$\psi = \frac{-y}{x^2 + y^2}$$ 10

   (b) Show that the bilinear transformation

   $w = \dfrac{(2z+3)}{(z-4)}$ maps the circle $x^2 + y^2 - 4x = 0$ into the line

   $4u + 3 = 0$ 10

Unit–III

5. (a) There are three bags : First containing 1 white, 2 red, 3 green balls; second 2 white, 3 red, 1 green balls and third contain 3 white, 1 red, 2 green balls. Two balls are drawn from an bag choosen at random. These are found to be one white and one red. Find the probability that the balls so drawn come from the second bag.   10

   (b) A function is defined as follows:

   $$f(x)\begin{cases} 0 & , & x < 2 \\[2mm] \dfrac{(2x+3)}{18} & , & 2 \le x \le 4 \\[2mm] 0 & , & x > 4 \end{cases}$$

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Show that it is a density function. Find the probability that a variate having this density will fall in the interval $2 \leq x \leq 3$?

10

6. (a) In 100 sets of ten tosses of an unbiased coin, in how many cases do you expect to get

    (i)   7 heads and 3 tails

    (ii) at least 7 heads?

    (iii) In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads and 4 tails?   10

    (b) If the mean height of an Indian police inspector be 170 cm with variance 25 cm^2, how many inspectors out of 1000 would you expect   10

    (i)   Between 170 cm and 180 cm,

    (ii) Less than 160 cm ?

Unit–IV

7. (a) Using Graphical method, solve the LPP,

    Maximize. $z = 2x_1 + 3x_2$, subject to the conditions

    $x_1 - x_2 \leq 2$; $x_1 + x_2 \geq 4$; $x_1, x_2 \geq 0$.

    (b) Define the following terme: 10

    (i)   In feasible solution

    (ii) Unbounded solution

    (iii) Multiple optimal solutions

    (iv) Redundancy.

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8. By Dual simplex method, solve the following LPP.

Maximize. $Z = 2x_1 + x_2$,

Subject to $3x_1 + x_2 \geq 3$;

$4x_1 + 3x_2 \geq 6$;

$3x_1 + 2x_2 \leq 3$;

$x_1, x_2 \geq 0$

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