

Roll No.

Total Pages : 3

BT-1/D-18

31047

CALCULUS AND LINEAR ALGEBRA

Paper : BS-133A

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *five* questions in all, selecting at least *one* question from each unit. All questions carry equal marks.

UNIT-I

1. (a) What is Gamma Function? Explain its reduction formula.
- (b) Find the volume formed by the revolution of loop of the curve $y^2(a + x) = x^2(3a - x)$ about the x -axis.
2. (a) State and prove Rolle's Theorem. If $f(x) = x(x-2)e^{\frac{3x}{4}}$ in $(0, 2)$ find c .

(b) Evaluate $\lim_{x \rightarrow 0} \frac{\left(1 + \frac{1}{x}\right)^x - e}{x}$.

UNIT-II

3. (a) Reduce the following matrix into normal form and hence find its rank :

$$\begin{bmatrix} 1 & -1 & -2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

- (b) Are the following vectors linearly independent ?
If so, find the relation between them : $(1, 1, 1, 3)$;
 $(1, 2, 3, 4)$; $(2, 3, 4, 9)$.
4. (a) Solve the following simultaneous equations by matrix inversion method.
 $x + y + z = 3$, $x + 2y + 3z = 4$, $2x + 3y + 4z = 7$.
- (b) Expressing matrix form, solve the following system of simultaneous equations using Gauss Jordan method.
 $2x_1 + x_2 + 2x_3 + x_4 = 6$, $6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$,
 $4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$ and $2x_1 + 2x_2 - x_3 + x_4 = 10$.

UNIT-III

5. (a) Why isn't \mathbb{R}^2 a subspace of \mathbb{R}^3 .
- (b) If u, v, w are independent vectors, show that the sums $v_1 = w_2 + w_3$, $v_2 = w_1 + w_3$ and $v_3 = w_1 + w_2$ are independent. (Write $c_1v_1 + c_2v_2 + c_3v_3 = 0$ in terms of the w 's. Find and solve equations for the c 's.

6. (a) Find the range and kernel (those are the new words for the column space and nullspace) of T when $T(v_1, v_2) = (v_1, v_2)$.
- (b) Suppose $T(v) = v$, except that $T(0, v_2) = (0, 0)$. Show that this transformation satisfies $T(cv) = cT(v)$ but not $T(u + v) = T(u) + T(v)$.

UNIT-IV

7. (a) Solve the system of equations
- $$\begin{aligned} x_1 + 2x_3 - 2x_4 &= 0, & 2x_1 - x_2 - x_4 &= 0, \\ x_1 + 2x_3 - x_4 &= 0, & 4x_1 - x_2 + 3x_3 - x_4 &= 0. \end{aligned}$$
- (b) (i) Modulus of each of the characteristic roots of a unitary matrix is unitary.
- (ii) Any square matrix can be expressed uniquely as a sum of Hermitian and Skew-Hermitian.

8. (a) Reduce the matrix $\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal

form and hence find its canonical form.

- (b) Verify Cayley Hamilton theorem for the matrix

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}. \text{ Hence compute } A^{-1}.$$